The Value-Undermining Effects of Rock Mining on Nearby Residential Property: A Semiparametric Spatial Quantile Autoregression*

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Abstract

Rock mining operations, including limestone and gravel production, have considerable adverse effects on residential quality of life due to elevated noise and dust levels resulting from dynamite blasting and increased truck traffic. This paper provides the first estimates of the effects of rock mining—an environmental disamenity—on local residential property values. We focus on the relationship between a house's price and its distance from nearby rock mine. Our analysis studies Delaware County, Ohio which, given its unique features, provides a natural environment for the valuation of property-value-suppressing effects of rock mines on nearby houses. We improve upon the conventional approach to valuating adverse effects of environmental disamenities based on hedonic house price functions. Specifically, in a pursuit of robust estimates, we develop a novel (semiparametric) partially linear spatial quantile autoregressive model which accommodates unspecified nonlinearities, distributional heterogeneity as well as spatial dependence in the data. We derive the consistency and normality limit results for our estimator as well as propose a consistent model specification test. We find statistically and economically significant property-value-suppressing effects of being located near an operational rock mine which gradually decline to insignificant near-zero values at a roughly ten-mile distance. Our estimates suggest that, all else equal, a house located a mile closer to a rock mine is priced, on average, at about 2.3–5.1% discount, with more expensive properties being subject to larger markdowns.

Keywords: Environmental Disamenity, Hedonic Model, Partially Linear, Quantile Regression, Rock Mines, SAR, Semiparametric, Spatial Lag

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1 Introduction

This paper provides the first estimates of the effects of rock mining—an environmental disamenity—on local residential property values. Rock mining operations, including limestone rock blasting and gravel mining, have considerable adverse effects on residential quality of life primarily due to elevated noise and dust levels resulting from blasting and increased truck traffic. Exacerbating matters, residential building activity and rock mining are also both pro-cyclical. Further, mining operations naturally seek to minimize their transportation costs by locating closer to their consumers in populated areas (Jaeger, 2006) thus increasing opportunities for opposition from local homeowners and citizen groups due to negative externalities associated with the former.

To valuate the effects of rock mining, we estimate Rosen's (1974) first-stage hedonic house price gradient which has long been used to estimate implicit prices of non-marketable local public goods or, as in our case, public bads from the housing market data. To this end, we focus on the relationship between a house's price and its distance from nearby rock mine. This distance effectively represents environmental amenity/quality, with better quality occurring at farther distances from mines as customarily presumed in hedonic studies. Our analysis focuses on Delaware County, Ohio which, given its unique features, provides a natural environment for the valuation of property-value-suppressing effects (if any) of rock mines on nearby houses. According to the U.S. Census Bureau, Delaware County has been among the two fastest growing counties in the state for the past twenty years. At the same time, given its geology, the county has rich limestone formations that have long been exploited as surface mines.[1] Consequently, residential and commercial expansion in the county has been in conflict with traditional land uses: farming and, especially, rock mining.

In our analysis, we seek to improve upon the conventional approach to valuating adverse effects of environmental disamenities based on hedonic house price functions. Specifically, in a pursuit of robust estimates of property-value effects of rock mines located in the vicinity of residential real estate, we estimate a house valuation function via novel (semiparametric) partially linear spatial quantile autoregressive model. The motivation for developing our model is threefold.

First, our partially linear model allows the distance from a house to nearby rock mine to enter the hedonic house price function in a completely unspecified nonparametric fashion thereby accommodating any potential nonlinearities in the relationship between property values and disamenity. This constitutes a significant improvement over prior studies most of which assume linearity and hence a constant marginal effect of the environmental disamenity on house prices. Few exceptions in earlier work include Harrison & Rubenfeld (1978), Kohlhase (1991), Leggett & Bockstael (2000), Hite et al. (2001), Cohen & Coughlin (2008) and Zabel & Guignet (2012) who model the disamenity quadratically, logarithmically or as a series of range-based dummy variables. In contrast to the latter studies, ours however does not assume the form of nonlinearity *a priori* and instead lets the data determine the nature of functional dependence between the distance to rock mine and house prices. Furthermore, by having the price of a house vary with its distance to mine nonparametrically, one no longer needs to *pre*specify the distance threshold beyond which the disamenity is presumed to have a zero effect on property values. Motivated by the argument that the effects of local disamenities are *local* in nature, the latter is usually done by fixing a spatial radius around a given disamenity thereby defining a circular area to be included in the analysis (e.g., Nelson et al., 1992; Reichert et al., 1992; Hite et al., 2001). In practice, the need to prespecify the radius is oftentimes dictated by the fact that one is more likely to find counterintuitive results if "irrelevant" data from far distances are included in the estimation of a parametric model that inherently cannot accommodate unknown nonlinearities in the property-value effects of disamenities, unless correctly

[1]Source: Ohio Department of Natural Resources.

prespecified. Our model is far more robust to this problem since it assumes no particular form of nonlinearity in the relationship between property values and disamenity.

Second, it is well-known in the real estate literature that environmental disamenities are likely to have heterogeneous impacts on residential property values with larger effects expected in more expensive upscale neighborhoods and more modest effects in less expensive areas (e.g., Reichert et al., 1992; Gayer, 2000). Nonetheless, virtually all earlier attempts at measuring the impact of environmental disamenities on property values have done so by estimating a hedonic house price function at the conditional *mean*. Such an approach delivers the marginal effect on the average house price, which can be rather uninformative from a policy perspective even after controlling for neighborhood characteristics because an "average" may not be representative of actual properties within the same locality, especially in the presence of thick tails of the house price distribution. In order to accommodate heterogeneous effects, we therefore assess the property-value impact of rock mines at different conditional *quantiles* of the house price distribution. We accomplish the latter by estimating a quantile regression model which, besides being more robust to the error distributions including the presence of outliers, allows for *distributional* heterogeneity of the effects of rock mines on property values.

Third, our model explicitly allows for spatial dependence in property values. By estimating a spatially autoregressive hedonic price function, we are able to indirectly control for *unobserved* neighborhood characteristics and shared local amenities (e.g., parks, playgrounds, traffic, air quality, crime, etc.) that affect property values. The spatial lag measuring the average price of neighboring houses serves as a good proxy for these unobserved neighborhood-wide attributes because, owing to their shared nature, they are also priced into the *observed* values of neighboring properties. While these characteristics can be partly controlled for using locality fixed effects, such an approach may be unsatisfactory since it does not let characteristics of neighboring houses affect the price of a given house (Anselin & Lozano-Gracia, 2009). However, by including the spatial lag in a hedonic house pricing function, we are able to accommodate such cross-neighbor effects as can be seen from a reduced form of our model whereby the conditional quantile of house price depends not only on its own attributes but also on its neighbors'. Perhaps more importantly, the spatial lag also contains information about (and thus can proxy for) unobserved *property*-specific attributes such as curb appeal because a given property's value, which is already reflective of its unobserved characteristics, affects its neighboring house's price through the "sales comparison approach" to a real estate appraisal whereby real estate agents base their appraisals of properties on the sale price information for houses in the neighborhood (see the references in Small & Steimetz, 2012). Thus, our spatially autoregressive hedonic model is significantly more robust to the omitted variable bias problem, which the overwhelming majority of housing-market-based valuations of adverse effects of environmental disamenities suffer from (Chay & Greenstone, 2005; Bajari et al., 2012). Prior papers that have also employed spatial hedonic models are largely limited to Gawande & Jenkins-Smith (2001), Brasington & Hite (2005) and Cohen & Coughlin (2008) although, unlike us, these studies of environmental disamenities focus on more restrictive parametric conditional mean models.

Our econometric model itself is a stand-alone contribution to the literature. It constitutes a practically useful fusion of semi/nonparametric quantile methods with models of spatial dependence. While the econometric literature has recently seen a rapid development in the theory of nonparametric estimation of quantile models (e.g., He & Shi, 1996; Yu & Jones, 1998; He & Liang, 2000; Lee, 2003; Honda, 2004; Kim, 2007), most such papers however do not allow endogenous explanatory variables as well as rule out any cross-sectional dependence by focusing on the case of *i.i.d.* data. In this paper, we consider quantile regression in the presence of endogeneity-inducing spatial dependence in the outcome variable. Our model nests several special cases that have been studied in the literature with Su & Yang (2011) and Su & Hoshino (2016) being the two most closely related papers [see Section 2 for more discussion]. Building on Chernozhukov & Hansen (2006), we propose estimating our model via a two-step nonparametric sieve instrumental variable (IV) quantile estimator. Under fairly mild regularity conditions, we show that our estimator is consistent and asymptotically normal. Furthermore, given that our partially linear model nests a more traditional *fully* linear spatial autoregressive model as a special case, one may naturally wish to formally discriminate between the two. To do so, we propose a bootstrap model specification test statistic which provides a vehicle for testing for a fully parametric specification of the spatial autoregression as well as an overall relevancy of some covariates in the model. The motivation for our test statistic comes from Ullah's (1985) nonparametric likelihood-ratio test formulated for a conditional mean model[2] which we extend to the quantile framework along the lines of Koenker & Machado (1999). We show the proposed is a consistent test.

We find statistically and economically significant property-value-suppressing effects of being located near an operational rock mine which gradually decline to insignificant near-zero values at a roughly ten-mile distance. For residential property in the middle of the price distribution, our estimates suggest that, all else equal, a house located a mile closer to a rock mine is predicted to be priced, on average, at about 3.1% discount. The analogous average discounts for houses in the first and third quartiles of price distribution are around 2.3 and 3.4%, respectively. For upscale property in the 0.95th quantile, it is at an astounding 5.1%. As a back-of-the-envelope welfare calculation, the above discount estimates imply the average loss in property value associated with the house being located a mile closer to a rock mine ranging from \$3,691 to \$10,970 for houses within the interquartile range of price distribution. For more expensive neighborhoods in the 0.95th quantile, such losses can be, on average, as high as \$28,410. Applying the estimated statistically significant discounts to house prices at each observation lying within a 10-mile radius from the mine to predict an increase in each property's value if it were moved from its actual location to a (counterfactual) 10-mile distance from the mine, we find the aggregate property value loss associated with rock mining in the area to be \$68.4 million at the median. Overall, using our specification test, we find that the proximity to rock mines *does* matter for residential property values.

The rest of the paper unfolds as follows. We first introduce our econometric model in Section 2, where we outline a two-step estimation methodology for it as well as provide its large-sample statistical properties. Section 3 presents a model specification test. (We study the finite-sample performance of our proposed estimator and the test statistic in a small set of Monte Carlo simulations in Appendix B.) We discuss the data in Section 4. The empirical results are reported in Section 5. Section 6 concludes.

2 A Partially Linear Spatial Quantile Autoregression

Following Jenish & Prucha (2012) and Qu & Lee (2015), we study spatial processes located on a (possibly) uneven lattice space $D \subseteq R^d$ for some $d \geq 1$. Let $\mathcal{Z}_n = \{(y_{i,n}, \mathbf{x}_{i,n}, \mathbf{z}_{i,n}, u_{i,n}, \varepsilon_{i,n}) : \boldsymbol{l}(i) \in D_n, n \geq 1\}$ be a triangular array of random fields defined on a probability space $(\Omega, \mathcal{F}, P)$ with $D_n \subset D$, where D_n is a finite subset of D, and $\boldsymbol{l}(i)$ refers to the location of the ith spatial unit in D, which is equipped with some distance metric $\varrho(i,j)$. For instance, we can let $\varrho(i,j) = \|\boldsymbol{l}(i) - \boldsymbol{l}(j)\|$ be a Euclidean distance between location $\boldsymbol{l}(i)$ and $\boldsymbol{l}(j)$. Also, let $|U|$ denote the cardinality of a finite subset $U \subset D$. We consider the increasing domain asymptotics as described in the following assumption.

[2] Also see Fan et al. (2001) and Lee & Ullah (2003).

Assumption 1 *The lattice D is infinitely countable with $|D_n| = n$, and $\varrho(i,j) > \varrho_0 > 0$ for any $i \neq j$.*

We consider the following PLSQAR model for a given quantile index τ:

$$y_{i,n} = \rho_{\tau,0} \sum_{j \neq i} w_{ij,n} y_{j,n} + \mathbf{x}_{i,n}' \boldsymbol{\beta}_{\tau,0} + \alpha_{\tau,0}(\mathbf{z}_{i,n}) + u_{i,n} \quad \forall\, \tau \in (0,1), \tag{2.1}$$

where $y_{i,n}$ is the (scalar) outcome variable of interest; $\mathbf{x}_{i,n}$ and $\mathbf{z}_{i,n}$ are $d_x \times 1$ and $d_z \times 1$ vectors of exogenous covariates, respectively; $\sum_{j \neq i} w_{ij,n} y_{j,n}$ is the endogeneity-inducing spatial lag with $w_{ij,n}$ being the (i,j)-th element of an $n \times n$ non-stochastic spatial weighting matrix $\mathbf{W}_n$ such that $w_{ii,n} = 0$ for all i and $\max_{1 \leq i \leq n} |\lambda_i \{\mathbf{W}_n\}| \leq 1$ where $\lambda_i\{\mathbf{A}\}$ is the ith eigenvalue of some $n \times n$ matrix $\mathbf{A}$; $\rho_{\tau,0} \in (-1,1)$ is a scalar varying spatial lag parameter function; $\boldsymbol{\beta}_{\tau,0}$ is a $d_x \times 1$ vector of constant slope parameters; and $\alpha_{\tau,0}(\cdot)$ is a scalar nonparametric function of $\mathbf{z}_{i,n}$. For identification purposes, $\mathbf{x}_{i,n}$ is assumed to include non-constant regressors only, and hence function $\alpha_{\tau,0}(\cdot)$ subsumes a traditional constant intercept parameter. Therefore, we refer to $\alpha_{\tau,0}(\cdot)$ as the "intercept function". Lastly, $u_{i,n}$ is the quantile error term such that

$$\Pr[u_{i,n} \leq 0 | \mathbf{X}_n, \mathbf{Z}_n, \mathbf{M}_n] = \tau \quad \text{a.s.} \quad \forall\, i = 1, \ldots, n, \tag{2.2}$$

where $\mathbf{X}_n = (\mathbf{x}_{1,n}, \ldots, \mathbf{x}_{n,n})'$ and $\mathbf{Z}_n = (\mathbf{z}_{1,n}, \ldots, \mathbf{z}_{n,n})'$ are $n \times d_x$ and $n \times d_z$ data matrices, respectively; and $\mathbf{M}_n = (\mathbf{m}_{1,n}, \ldots, \mathbf{m}_{n,n})'$ is an $n \times d_m$ instrument matrix with $\mathbf{m}_{i,n}$ being a $d_m \times 1$ vector of valid instruments for the endogenous spatial lag $\sum_{j \neq i} w_{ij,n} y_{j,n}$.

Letting $\mathbf{y}_n = (y_{1,n}, \ldots, y_{n,n})'$ and $\mathbf{u}_n = (u_{1,n}, \ldots, u_{n,n})'$, we can rewrite our model (2.1) in the matrix form as follows

$$\mathbf{y}_n = \rho_{\tau,0} \mathbf{W}_n \mathbf{y}_n + \mathbf{X}_n \boldsymbol{\beta}_{\tau,0} + \boldsymbol{\alpha}_{\tau,0}(\mathbf{Z}_n) + \mathbf{u}_n, \tag{2.3}$$

where $\boldsymbol{\alpha}_{\tau,0}(\mathbf{Z}_n) = (\alpha_{\tau,0}(\mathbf{z}_{1,n}), \ldots, \alpha_{\tau,0}(\mathbf{z}_{n,n}))'$. From (2.3), it is evident that, by assuming that the eigenvalues of $\mathbf{W}_n$ do not exceed one in absolute magnitude[3] and that the spatial lag parameter lies within the unit circle, we ensure the non-singularity of $\mathbf{I}_n - \rho_{\tau,0} \mathbf{W}_n$ necessary to guarantee the existence of the reduced form for our model:

$$\mathbf{y}_n = [\mathbf{I}_n - \rho_{\tau,0} \mathbf{W}_n]^{-1} \left(\mathbf{X}_n \boldsymbol{\beta}_{\tau,0} + \boldsymbol{\alpha}_{\tau,0}(\mathbf{Z}_n) + \mathbf{u}_n \right). \tag{2.4}$$

The appeal of our proposed semiparametric PLSQAR model in (2.1) is at least two-fold. First, not only does it accommodate heterogeneity in the spatial relationship by allowing some covariates in the model (namely, $\mathbf{z}_{i,n}$) to affect the outcome variable in a completely unspecified way thereby admitting any potential unit-specific nonlinearities but it also allows for *distributional* heterogeneity of the effects of $\mathbf{X}_n$ and $\mathbf{Z}_n$ on $\mathbf{y}_n$. The latter is accomplished by separate measurements of the spatial relationship at different points of a response distribution. Second, unlike more conventional conditional mean models of spatial dependence, our quantile model is more robust to the error distributions including the presence of outliers.

Model (2.1) nests several special cases of quantile regressions that have been studied in the literature. Perhaps, the two most closely related models are those by Su & Yang (2011) and Su & Hoshino (2016). Specifically, if nonparametric intercept function $\alpha_{\tau,0}(\cdot)$ does not vary with $\mathbf{z}_{i,n}$ and is constant for any given quantile index τ, i.e., when $\alpha_{\tau,0}(\mathbf{z}_{i,n}) = \alpha_{\tau,0}$ for all $\mathbf{z}_{i,n}$, our model becomes a (more restrictive) *fully* parametric linear spatial quantile autoregression (SQAR) considered by

[3]Which is satisfied if one standardizes a raw spatial weighting matrix by dividing all of its elements by its largest eigenvalue in absolute value.

Su & Yang (2011). On the other hand, our model can also be viewed as a special case of Su & Hoshino's (2016) varying-coefficient quantile regression where all parameter functions, except for the intercept, are forced to be constant. However, while their model also features endogenous regressors, it rules out any cross-sectional dependence by focusing on the case of *i.i.d.* data. In contrast, our PLSQAR model relaxes the *i.i.d.* assumption by allowing the spatial dependence in $\mathbf{y}_n$. In the case when the outcome variable exhibits no spatial dependence and hence $\rho_{\tau,0} = 0$, our model is no longer subject to endogeneity and essentially becomes an ordinary partially linear quantile regression which has been rather extensively studied for *i.i.d.* data (e.g., He & Shi, 1996; He & Liang, 2000; Lee, 2003). If one further restricts $\boldsymbol{\beta}_{\tau,0} = \mathbf{0}_{d_x}$, the model collapses to a fully nonparametric quantile regression studied by Yu & Jones (1998). In case of exogenous regressors only, some other closely related models include a varying coefficient quantile regression studied by Honda (2004) and Kim (2007) for *i.i.d.* data and Cai & Xu (2008) for the time-series case.

2.1 Sieve IV Quantile Estimator

Our estimation strategy relies on Chernozhukov & Hansen's (2006) idea whereby the solution to the instrument-based quantile restriction (2.2) is essentially equivalent to the search for $\left(\rho_{\tau,0}, \boldsymbol{\beta}'_{\tau,0}, \alpha_{\tau,0}(\mathbf{z}_{i,n})\right)'$ such that zero is the solution to the usual quantile regression of $y_{i,n} - \rho_{\tau,0}\sum_{j\neq i} w_{ij,n}y_{j,n} - \mathbf{x}'_{i,n}\boldsymbol{\beta}_{\tau,0} - \alpha_{\tau,0}(\mathbf{z}_{i,n})$ on exogenous $(\mathbf{x}_{i,n}, \mathbf{z}_{i,n}, \mathbf{m}_{i,n})$, i.e.,

$$0 \in \arg\min_{f\in\mathcal{H}} \mathbb{E}\left[\zeta_\tau\left\{\left(y_{i,n} - \rho_{\tau,0}\sum_{j\neq i} w_{ij,n}y_{j,n} - \mathbf{x}'_{i,n}\boldsymbol{\beta}_{\tau,0} - \alpha_{\tau,0}(\mathbf{z}_{i,n})\right) - f(\mathbf{x}_{i,n}, \mathbf{z}_{i,n}, \mathbf{m}_{i,n})\right\}\right], \tag{2.5}$$

where $\zeta_\tau\{u\} \equiv u(\tau - \mathbb{1}\{u<0\})$ for some $u \in R$ is the so-called "check function" with $\mathbb{1}\{\cdot\}$ being the indicator function, and $f(\cdot) \in \mathcal{H}$ is some measurable function.

Chernozhukov & Hansen (2006) pioneered this "instrumental variable quantile regression" approach for a parametric (fully linear) constant-coefficient model. Recently, it has been extended to a broader class of semiparametric varying-coefficient models by Su & Hoshino (2016). Both papers however assume *i.i.d.* data, which is certainly *not* the case in our paper given the spatial dependence in $\mathbf{y}_n$. We show that, under some regularity conditions, the approach nonetheless remains valid even for the spatial data. Different from Su & Yang (2011) who study the fully parametric special case of our model, we do so using the Law of Large Numbers (LLN) and Central Limit Theorem (CLT) for spatial near-epoch dependent (NED) processes derived in Jenish & Prucha (2012). In what follows, we outline the estimation methodology for our PLSQAR model. The asymptotic results along with the necessary assumptions to support them are discussed in Section 2.2.

We approximate unknown nonparametric function using sieves [for an excellent review of the sieve methods, see Chen (2007)]. Specifically, let $\{\phi_1(\cdot), \phi_2(\cdot), \ldots\}$ be a sequence of B-spline series (or the tensor product thereof). Then, for each z, we approximate the unknown intercept function $\alpha_{\tau,0}(z)$ by $\boldsymbol{\phi}_{L_n}(z)'\mathcal{A}_{\tau,0}$ where, for any integer $\kappa > 0$, we denote a $\kappa \times 1$ vector of known basis functions $\boldsymbol{\phi}_\kappa(u) = (\phi_1(u), \ldots, \phi_\kappa(u))'$, and the unknown parameter vector $\mathcal{A}_{\tau,0}$ is of dimension L_n. Hence, we can now rewrite our model in (2.1) as follows

$$y_{i,n} \approx \rho_{\tau,0}\sum_{j\neq i} w_{ij,n}y_{j,n} + \mathbf{x}'_{i,n}\boldsymbol{\beta}_{\tau,0} + \boldsymbol{\phi}_{L_n}(\mathbf{z}_{i,n})'\mathcal{A}_{\tau,0} + u_{i,n} \quad \forall\, \tau \in (0,1). \tag{2.6}$$

Following Chernozhukov & Hansen (2006), we also restrict $\mathcal{H}$ to the following class of linear functions:

$$\mathcal{H} = \left\{f(\mathbf{x}_{i,n}, \mathbf{z}_{i,n}, \mathbf{m}_{i,n}) = \mathbf{m}'_{i,n}\gamma\right\}, \tag{2.7}$$

where $\boldsymbol{\gamma}$ is a $d_m \times 1$ vector of constant parameters.

The sample counterpart of the objective function in the population instrumental variable quantile regression (2.5) then takes the following form:

$$\mathbb{Q}_{n,\tau}(\rho, \boldsymbol{\beta}, \mathcal{A}, \boldsymbol{\gamma}) \equiv \frac{1}{n} \sum_{i=1}^{n} \zeta_\tau \left\{ y_{i,n} - \rho \sum_{j \neq i} w_{ij,n} y_{j,n} - \mathbf{x}_{i,n}' \boldsymbol{\beta} - \boldsymbol{\phi}_{L_n} (\mathbf{z}_{i,n})' \mathcal{A} - \mathbf{m}_{i,n}' \gamma \right\}. \tag{2.8}$$

Based on the rationale behind (2.5), one is to expect the estimate of $\boldsymbol{\gamma}_\tau$ to be close to zero when the estimate of $\left(\rho_{\tau,0}, \boldsymbol{\beta}_{\tau,0}', \alpha_{\tau,0}(\cdot)\right)'$ is close to the true population value. Building on this intuition, we can estimate unknown $\left(\rho_{\tau,0}, \boldsymbol{\beta}_{\tau,0}', \alpha_{\tau,0}(\cdot)\right)'$ in two steps.

Step 1. For a given value of ρ, we estimate the usual quantile regression of $\dot{y}_{i,n}(\rho) \equiv y_{i,n} - \rho \sum_{j \neq i} w_{ij,n} y_{j,n}$ on exogenous covariates $\mathcal{X}_{i,n} = \left(\mathbf{x}_{i,n}', \mathbf{m}_{i,n}', \boldsymbol{\phi}_{L_n}(\mathbf{z}_{i,n})'\right)'$ to obtain the "profiled" estimates of $\boldsymbol{\theta}_{\tau,0}(\rho) = \left(\boldsymbol{\beta}_{\tau,0}(\rho)', \boldsymbol{\gamma}_{\tau,0}(\rho)', \mathcal{A}_{\tau,0}(\rho)'\right)'$:

$$\widehat{\boldsymbol{\theta}}_\tau(\rho) = \underset{\boldsymbol{\theta}(\rho) \in \boldsymbol{\Theta}}{\arg\min} \ \frac{1}{n} \sum_{i=1}^{n} \zeta_\tau \left\{ \dot{y}_{i,n}(\rho) - \mathcal{X}_{i,n}' \boldsymbol{\theta}(\rho) \right\}, \tag{2.9}$$

where $\boldsymbol{\theta}_{\tau,0}(\rho)$ is an interior point of $\boldsymbol{\Theta}$, a compact subset of $R^{1+d_x+d_m+L_n}$, and is the unique solution to the population counterpart of (2.9):

$$\boldsymbol{\theta}_{\tau,0}(\rho) = \underset{\boldsymbol{\theta}_0(\rho) \in \boldsymbol{\Theta}}{\arg\min} \ \mathbb{E}\left[\zeta_\tau \left\{ \dot{y}_{i,n}(\rho) - \mathcal{X}_{i,n}' \boldsymbol{\theta}_0(\rho) \right\}\right]. \tag{2.10}$$

Step 2. We minimize the weighted norm of $\widehat{\boldsymbol{\gamma}}_\tau(\rho)$ estimated in the first step with respect to ρ to obtain our estimator of $\rho_{\tau,0}$:

$$\widehat{\rho}_\tau = \underset{\rho}{\arg\min} \ \widehat{\boldsymbol{\gamma}}_\tau(\rho)' \mathbf{V}_n \widehat{\boldsymbol{\gamma}}_\tau(\rho), \tag{2.11}$$

where $\mathbf{V}_n$ is some $d_m \times d_m$ symmetric positive-definite weighting matrix. Correspondingly, the estimators of $\boldsymbol{\beta}_{\tau,0}$ and $\mathcal{A}_{\tau,0}$ are respectively given by

$$\widehat{\boldsymbol{\beta}}_\tau = \widehat{\boldsymbol{\beta}}_\tau(\widehat{\rho}_\tau) \quad \text{and} \quad \widehat{\mathcal{A}}_\tau = \widehat{\mathcal{A}}_\tau(\widehat{\rho}_\tau). \tag{2.12}$$

Hence, for any given $\mathbf{z}$, the sieve estimator of the unknown intercept function $\alpha_{\tau,0}(\mathbf{z})$ is

$$\widehat{\alpha}_\tau(\mathbf{z}) = \boldsymbol{\phi}_{L_n}(\mathbf{z})' \widehat{\mathcal{A}}_\tau. \tag{2.13}$$

The implementation of our estimator warrants three remarks. First, assuming that $\mathbf{x}_{i,n}$ and $\mathbf{z}_{i,n}$ are strictly exogenous and relevant, a selection of linearly independent variables from $\mathbf{W}_n\mathbf{X}_n$, $\mathbf{W}_n\mathbf{Z}_n$, $\mathbf{W}_n^2\mathbf{X}_n$, $\mathbf{W}_n^2\mathbf{Z}_n, \ldots$ provides a set of good instruments for the endogenous spatial lag $\mathbf{W}_n\mathbf{y}_n$. Since we only seek to obtain a consistent nonparametric IV estimator without pursuing optimality, we use $\mathbf{m}_{i,n} = \left[(\mathbf{W}_n\mathbf{X}_n)_i', (\mathbf{W}_n\mathbf{Z}_n)_i'\right]'$ as our instruments, having removed any redundant terms, where $(\mathbf{W}_n\mathbf{A})_i = \sum_{j \neq i} w_{ij,n} a_j$ for $\mathbf{A} = \mathbf{X}_n, \mathbf{Z}_n$. Second, the outlined two-step estimation methodology can be operationalized in the form of a grid search or, alternatively, both steps can be estimated jointly via an automatic numerical search. In either case, it is imperative to impose appropriate box constraints on ρ to ensure that it lies within the unit circle. Third, in the second-step estimation, an obvious practical choice for $\mathbf{V}_n$ is an identity matrix, as suggested by Chernozhukov & Hansen (2006) and Su & Yang (2011). In fact, when $d_m = 1$ and our model is exactly identified, we can show that the limiting distribution of our estimator is expectedly invariant to the choice of $\mathbf{V}_n$. In the case of an over-identified model, one however could improve asymptotic efficiency by weighing $\widehat{\boldsymbol{\gamma}}_\tau(\rho)$ using the inverse of its asymptotic covariance matrix, which obviously would first need to be consistently estimated. For tractability purposes, in our paper we set $\mathbf{V}_n = \mathbf{I}_{d_m}$.

2.2 Asymptotic Properties

The derivation of limit results for our proposed estimator requires the following assumptions.

Assumption 2 *(i)* $\{(\mathbf{x}_{i,n}, \mathbf{z}_{i,n})\}$ *is non-stochastic and uniformly bounded in absolute values; (ii)* $u_{i,n} = b_{i,n}(\mathbf{X}_n, \mathbf{Z}_n, \boldsymbol{\varepsilon}_n)$ *is a function of* $\mathbf{X}_n$*,* $\mathbf{Z}_n$ *and* $\boldsymbol{\varepsilon}_n$ *such that* $\Pr(u_{i,n} \le 0) = \tau$ *holds almost surely for all i, and* $\boldsymbol{\varepsilon}_n = (\varepsilon_{1,n}, \ldots, \varepsilon_{n,n})$ *is an* $n \times 1$ *vector of errors with uniformly bounded variances; (iii)* $\{u_{i,n}, \boldsymbol{l}(i) \in D_n\}$ *is uniformly* L_2*-NED on* $\{\varepsilon_{j,n}, \boldsymbol{l}(i) \in D_n\}$ *with the NED coefficients of* $\psi(s) = O(s^{-\varsigma})$ *for some* $\varsigma > d$*, and the* α*-mixing coefficients of* $\{\varepsilon_{i,n}\}$ *satisfy* $\alpha(k,l,r) \le (k+l)^{\upsilon}\,\widehat{\alpha}(r)$ *for some* $\upsilon \ge 0$ *and* $\sum_{r=1}^{\infty} r^{d(\upsilon+1)-1}\widehat{\alpha}(r) < \infty$*, where the NED concept is defined over* $\mathcal{F}_{i,n}(s) = \sigma(\varepsilon_{j,n}, \boldsymbol{l}(j) \in D_n, \varrho(i,j) \le s)$*, the smallest* σ*-field generated by* $\{\varepsilon_{i,n}\}$ *located in the s-neighborhood of the spatial unit i.*

Assumption **2(i)**, also used by Qu & Lee (2015), permits a simple exposition of our assumptions without loss of generality and can be relaxed to allow stochasticity with bounded moment conditions. Under Assumption **2(ii)–(iii)**, $\{u_{i,n}, \boldsymbol{l}(i) \in D_n\}$ is a weakly dependent spatial process with heteroskedasticity. To conserve space, we refer the reader to Jenish & Prucha (2009, 2012) for definition of the spatial α-mixing and NED process including $\alpha(k,l,r)$ and $\widehat{\alpha}(r)$. Since $\mathbf{X}_n$ and $\mathbf{Z}_n$ are non-stochastic, the stochastic property of $u_{i,n}$ is determined solely by its location $\boldsymbol{l}(i)$ and a nonlinear moving average of $\boldsymbol{\varepsilon}_n$. According to Jenish & Prucha (2012), Assumption **2(iii)** holds if $\max_{1\le i\le n} \mathbb{E}[\varepsilon_{i,n}^2] < M < \infty$ and the overall contributions (i.e., weights) of $\{\varepsilon_{i,n}\}$ in absolute values are ignorable among far-away spatial units. The convergence speeds of the mixing coefficients and the NED coefficients to zero are the same as those in Jenish (2016).

To see the validity of Assumption **2(iii)**, consider an example of $u_{i,n} = \sigma_{i,n}\varepsilon_{i,n}$, where $\{\varepsilon_{i,n}\}$ is an *i.i.d.* error with finite variance and $\sigma_{i,n} = \lambda_0 + \lambda_1 \sum_{j\ne i} w_{ij,n} y_{j,n} + \mathbf{x}_{i,n}'\boldsymbol{\lambda}_2 + \lambda_3(\mathbf{z}_{i,n})$. Combining with (2.3)–(2.4), we have that

$$\boldsymbol{\sigma}_n = \lambda_0 \mathbf{i}_n + \mathbf{X}_n\boldsymbol{\lambda}_2 + \boldsymbol{\lambda}_3(\mathbf{Z}_n) + \lambda_1\mathbf{G}_n\mathbf{X}_n\boldsymbol{\beta}_{\tau,0} + \lambda_1\mathbf{G}_n\boldsymbol{\alpha}_{\tau,0}(\mathbf{Z}_n) + \lambda_1\mathbf{G}_n\mathcal{E}_n\boldsymbol{\sigma}_n, \tag{2.14}$$

where $\boldsymbol{\sigma}_n = (\sigma_{1,n}, \ldots, \sigma_{n,n})'$, $\mathcal{E}_n = \operatorname{diag}\{\varepsilon_{1,n}, \ldots, \varepsilon_{n,n}\}$, and $\mathbf{i}_n$ is an $n \times 1$ vector of ones. Furthermore, letting $\mathbf{S}_n(\rho) = \mathbf{I}_n - \rho\mathbf{W}_n$ and $\mathbf{G}_n(\rho) = \mathbf{W}_n\mathbf{S}_n(\rho)^{-1}$, we define $\mathbf{S}_n = \mathbf{S}_n(\rho_{\tau,0})$ and $\mathbf{G}_n = \mathbf{G}_n(\rho_{\tau,0})$ the latter of which has a typical element $g_{ij,n}$. If the random matrix $\mathbf{I}_n - \lambda_1\mathbf{G}_n\mathcal{E}_n$ is invertible almost surely,[4] $\sigma_{i,n}$ is an MA(∞) spatial process of $\{\varepsilon_{i,n}\}$. Roughly speaking, $\{\sigma_{i,n}, \boldsymbol{l}(i) \in D_n\}$ is L_2-NED on $\{\varepsilon_{j,n}, \boldsymbol{l}(i) \in D_n\}$ by Proposition 1 in Jenish & Prucha (2012) if $\lim_{s\to\infty}\sup_{n,\boldsymbol{l}(i)\in D_n} \sum_{\boldsymbol{l}(j)\in D_n, \varrho(i,j)>s} |g_{ij,n}| = 0$. Consequently, $\{u_{i,n}, \boldsymbol{l}(i) \in D_n\}$ is L_2-NED on $\{\varepsilon_{j,n}, \boldsymbol{l}(i) \in D_n\}$.

[4]Let $e(\mathbf{A})$ be the largest eigenvalue of $\mathbf{A}$ in the absolute value, where $\mathbf{A}$ is an $n \times n$ matrix with a typical element a_{ij}. Then, $e(\mathbf{A}) \le \|\mathbf{A}\|_1$, where $\|\mathbf{A}\|_1 = \max_{1\le j\le n}\sum_{i=1}^n |a_{ij}|$ by Seber (2008, Property 4.68). Now, $\|\mathbf{I}_n - \lambda_1\mathbf{G}_n\mathcal{E}_n\|_1 \le |1 - \lambda_1 g_{jj,n}\varepsilon_{j,n}| + |\lambda_1| \max_{1\le j\le n}\sum_{i\ne j}|g_{ij}|\,|\varepsilon_{jn}| < 1$ holds almost surely if $\|\mathbf{G}_n\|_1 < M < \infty$, $\{\varepsilon_{in}\}$ has a compact support, and λ_1 is small enough (Seber, 2008, p.472), where $\|\mathbf{G}_n\|_1 < M < \infty$ is a regularity assumption commonly imposed in the spatial autoregressive literature (e.g., Kelejian & Prucha, 2010).

Assumption 3 *(i)* $\mathbf{S}_n(\rho)$ *is a nonsingular matrix over* $\rho \in \Lambda_\rho$, *and* $\rho_{\tau,0}$ *is an interior point of* Λ_ρ, *a compact subset of R; (ii) there exists a positive integer* N *such that both* $\mathbf{W}_n$ *and* $\mathbf{S}_n^{-1}(\rho)$ *have finite row- and column-sum matrix norms for all* $n > N$ *and* $\rho \in \Lambda_\rho$; *(iii)* $|w_{ij,n}| \leq c_1 \varrho(i,j)^{-c_2 d}$ *for some positive constants* c_1 *and* $c_2 > \varsigma/d$.

Assumption **3(i)–(ii)** are the regularity conditions (e.g., Kelejian & Prucha, 2010). Assumption **3(iii)** deviates from Qu & Lee (2015) by assuming gradually decaying spatial weights as the distance between two spatial units grows, which includes the case when $|w_{ij,n}| = 0$ if $\varrho(i,j)$ is greater than some threshold value.

Assumption 4 *(i) There exists an* $L_n \times 1$ *vector* $\mathcal{A}_{\tau,0}$ *such that*

$$\sup_{\mathbf{z} \in \mathcal{S}_\mathbf{z}} \left| \alpha_\tau(\mathbf{z}) - \mathcal{A}'_{\tau,0} \boldsymbol{\phi}_{L_n}(\mathbf{z}) \right| \leq M L_n^{-\xi} \tag{2.15}$$

for any $\rho \in \Lambda_\rho$ *and some* $\xi > 2$ *as* $L_n \to \infty$; *(ii)* $\{\phi_l(\cdot)\}$ *is uniformly bounded over all* l *such that* $\left\|\boldsymbol{\phi}_{L_n}\right\| = \sup_\mathbf{z} \sqrt{\sum_{l=1}^{L_n} \phi_l(\mathbf{z})} = O\left(\sqrt{L_n}\right)$.

Since $\mathcal{S}_z$ is a compact set, B-spline tensors can be used to construct the basis functions. Hence, Assumption **4** holds if $\alpha_\tau(\cdot)$ is p-smooth with uniformly bounded derivatives up to order p for some $p > \xi$.

Assumption 5 *Define* $\mathbf{v}_n(\rho) = [\mathbf{I}_n + (\rho_{\tau,0} - \rho)\mathbf{G}_n]\mathbf{u}_n$ *and let* $v_{i,n}(\rho)$ *be its ith element. (i)* $v_{i,n}(\rho)$ *has cdf* $F_{v_{i,n}(\rho)}(v)$ *and pdf* $f_{v_{i,n}(\rho)}(v)$, *and* $f_{v_{i,n}(\rho)}(v)$ *is continuously differentiable and uniformly bounded up to its first derivative with respect to* $v \in R$ *and* $\rho \in \Lambda_\rho$; *(ii) there exists two finite constants* $\underline{c}$ *and* $\bar{c}$ *such that* $0 < \underline{c} \leq \lambda_{\min}\{\boldsymbol{\Sigma}_\tau(\rho)\} \leq \lambda_{\max}\{\boldsymbol{\Sigma}_\tau(\rho)\} \leq \bar{c} < \infty$ *uniformly over* $\rho \in \Lambda_\rho$; *(iii)* $\mathcal{A}_2$ *is a nonsingular matrix, where* $\boldsymbol{\Sigma}_\tau(\rho)$ *and* $\mathcal{A}_2$ *are respectively defined in* (A.6) *and* (A.9) *in Appendix A.*

Since $v_{i,n}(\rho)$ is a linear combination of $\{u_{i,n}\}$, applying our earlier arguments and under Assumptions **2–3**, in Lemma 1 in Appendix A we show that $\{v_{i,n}(\rho), \boldsymbol{l}(i) \in D_n\}$ is also an L_2-NED on $\{\varepsilon_{i,n}, \boldsymbol{l}(i) \in D_n\}$ with the NED coefficients of $\psi(s) = O(s^{-\varsigma})$. Assumption **5(ii)** ensures the existence of the estimator calculated in Step 1, while Assumption **5(iii)** ensures the existence of the second-step estimator.

Assumption 6 *As* $n \to \infty$, $L_n \to \infty$, $nL_n^{1-2\xi} \to 0$ *and* $L_n^2/n \to 0$.

Assumption **6** is an assumption on the smoothing parameter L_n to ensure the consistency of our proposed estimator. Specifically, letting $L_n = cn^q$ for some $c > 0$Assumption **6** implies that $0 < 1/(2\xi - 1) < q < 1/2$.

Assumption 7 $F_{u_{i,n}}(u|\bar{u}_{i,n})$ *and* $f_{u_{i,n}}(u|\bar{u}_{i,n})$ *are, respectively, conditional cdf and pdf of* $u_{i,n} = u$ *conditional on* $\bar{u}_{i,n} = \sum_{j \neq i} g_{ij,n} u_{j,n}$, *and* $f_{u_{i,n}}(u|\bar{u}_{i,n})$ *is uniformly bounded and continuous up to the second-order derivatives with respect to u.*

Assumptions **1–6** are used to show the consistency of our first-step estimator, whereas Assumption **7** is used to derive the asymptotic normality results of the second-step estimator.

Theorem 1 *Under Assumptions **1–6**, we have that* $\max_{\rho \in \Lambda_\rho} \left\| \widehat{\boldsymbol{\theta}}_\tau(\rho) - \boldsymbol{\theta}_{\tau,0}(\rho) \right\| = O_p\left(\sqrt{L_n/n}\right)$.

Theorem 2 *Under Assumptions **1–7**, we have*

$$\sqrt{n}\boldsymbol{\Sigma}_n^{-1/2}\begin{pmatrix}\widehat{\rho}_\tau-\rho_{\tau,0}\\ \widehat{\boldsymbol{\beta}}_\tau-\boldsymbol{\beta}_{\tau,0}\\ \widehat{\gamma}_\tau\end{pmatrix}\overset{d}{\to}\mathbb{N}\left(\mathbf{0},\mathbf{I}_{1+d_x+d_m}\right)\quad and\quad \sqrt{n/\omega_{n,\tau}}\Big(\widehat{\alpha}_\tau\left(\mathbf{z}\right)-\alpha_{\tau,0}\left(\mathbf{z}\right)\Big)\overset{d}{\to}\mathbb{N}\left(0,1\right),$$

where $\boldsymbol{\Sigma}_n$ and $\omega_{n,\tau}$ are defined in the proof of this theorem in Appendix A.

From the proof of this theorem, we see that $\boldsymbol{\Sigma}_n$ is a nonsingular matrix under Assumption **5(ii)–(iii)** and that $\omega_{n,\tau}=O\left(\sqrt{L_n}\right)$.

Remark 1. We study the finite-sample performance of our proposed two-step estimator in a small set of Monte Carlo simulations, the discussion of which is relegated to Appendix B. Overall, the results are encouraging, and simulation experiments support our asymptotic results.

3 Specification Testing

We next consider a model specification test which permits testing several useful hypotheses. Specifically, for a τth spatial quantile autoregression written as

$$y_{i,n}=q\left(\sum_{j\neq i}w_{ij,n}y_{j,n},\mathbf{x}_{i,n},\mathbf{z}_{i,n},\tau\right)+u_{i,n}\equiv q_i(\tau)+u_{i,n},\tag{3.1}$$

we consider the following null hypotheses about the form of its conditional quantile function $q_i(\tau)$:

$$\mathrm{H}_0(\mathrm{i}):\quad q_i(\tau)=\rho_{\tau,0}\sum_{j\neq i}w_{ij,n}y_{j,n}+\mathbf{x}_{i,n}'\boldsymbol{\beta}_{\tau,0}+(1,\mathbf{z}_{i,n})'\boldsymbol{\delta}_{\tau,0}\tag{3.2}$$

$$\mathrm{H}_0(\mathrm{ii}):\quad q_i(\tau)=\rho_{\tau,0}\sum_{j\neq i}w_{ij,n}y_{j,n}+\mathbf{x}_{i,n}'\boldsymbol{\beta}_{\tau,0}+\delta_{\tau,0},\tag{3.3}$$

against the alternative (the PLSQAR model):

$$\mathrm{H}_1:\quad q_i(\tau)=\rho_{\tau,0}\sum_{j\neq i}w_{ij,n}y_{j,n}+\mathbf{x}_{i,n}'\boldsymbol{\beta}_{\tau,0}+\alpha_{\tau,0}(\mathbf{z}_{i,n}).\tag{3.4}$$

Alternatively, the above null and alternative hypotheses can be rewritten as follows: $\mathrm{H}_0(\mathrm{i})$: $\Pr\left[\alpha_{\tau,0}(\mathbf{z}_{i,n})=(1,\mathbf{z}_{i,n})'\boldsymbol{\delta}_{\tau,0}\right]=1$ for some $\boldsymbol{\delta}_{\tau,0}\in R^{1+d_z}$ against H_1 : $\Pr\left[\alpha_{\tau,0}(\mathbf{z}_{i,n})=(1,\mathbf{z}_{i,n})'\boldsymbol{\delta}_{\tau}\right]<1$ for any $\boldsymbol{\delta}_\tau\in R^{1+d_z}$, and $\mathrm{H}_0(\mathrm{ii})$: $\Pr\left[\alpha_{\tau,0}(\mathbf{z}_{i,n})=\delta_{\tau,0}\right]=1$ for some $\delta_{\tau,0}\in R$ against H_1 : $\Pr\left[\alpha_{\tau,0}(\mathbf{z}_{i,n})=\delta_{\tau}\right]<1$ for any $\delta_\tau\in R$. The null in (3.2) is meant to test for linearity of the conditional quantile function in $\mathbf{z}_{i,n}$. In practice, one may choose any desired *parametric* specification for the intercept function $\alpha_{\tau,0}(\cdot)$ to test against the nonparametric alternative in (3.4). The second null in (3.3) is essentially the test of overall relevancy of $\mathbf{z}_{i,n}$.

To test these hypotheses, we essentially propose a nonparametric likelihood-ratio test based on the comparison of the restricted and unrestricted models. The motivation for our test statistic comes from Ullah's (1985) nonparametric test that compares residual sums of squares under the null and the alternative (also see Fan et al., 2001; Lee & Ullah, 2003). The idea behind this test, which is formulated for a conditional mean model, can be extended to the conditional quantile framework along the lines of Koenker & Machado (1999) whereby the estimated residual sum of check functions effectively plays the role of the residual sum of squares. Specifically, for any given quantile index τ, we consider the following residual-based test statistic:

$$T_n = \frac{RSC_{0,\tau} - RSC_{1,\tau}}{RSC_{1,\tau}}, \tag{3.5}$$

where $RSC_{0,\tau}$ is the residual sum of check functions under H_0 computed as $RSC_{0,\tau} = \sum_{i=1}^{n} \zeta_\tau\{\widetilde{u}_{i,n}\}$ with $\widetilde{u}_{i,n} = y_{i,n} - \widetilde{q}_i(\tau)$ being the quantile residual defined as the difference between $y_{i,n}$ and the consistent estimate of $q_i(\tau)$ under either of the two null hypotheses in (3.2)–(3.3); and $RSC_{1,\tau}$ is the residual sum of check functions under H_1 computed as $RSC_{1,\tau} = \sum_{i=1}^{n} \zeta_\tau\{\widehat{u}_{i,n}\}$, where $\widehat{u}_{i,n}$ is the residual from our second-step estimator, i.e., $\widehat{u}_{i,n} = y_{i,n} - \widehat{q}_i(\tau) = y_i - \widehat{\rho}_\tau \sum_{j\neq i} w_{ij,n} y_{j,n} - \mathbf{x}'_{i,n}\widehat{\boldsymbol{\beta}}_\tau - \widehat{\alpha}_\tau(\mathbf{z}_{i,n})$. Residuals under H_0 can be obtained via Su & Yang's (2011) estimator.

Theorem 3 *Under Assumptions 2–5, under H_0 we have that $T_n \xrightarrow{p} 0$, while under H_1 we have $\Pr[T_n \geq M_n] \to 0$ for any non-stochastic, positive sequence M_n.*

See Appendix A for the proof. Thus, T_n is a consistent test. Intuitively, the test statistic is expected to converge to zero under the null and is positive under the alternative. Hence, the test is one-sided. We suggest using bootstrap for approximating the null distribution of T_n, especially given that residual-based nonparametric tests are well-known to perform rather poorly in finite samples when relying on asymptotic critical values. Bootstrap methods however offer a means to improve their finite-sample performance. For fixed $\tau \in (0, 1)$, we use the following wild (residual) bootstrap procedure modified to suit the asymmetric loss function used in the quantile estimation:[5]

(1) Estimate the restricted model under either of the two nulls in (3.2)–(3.3) to obtain residuals $\{\widetilde{u}_{i,n};\ i = 1, \ldots, n\}$.

(2) Generate two-point wild bootstrap errors by setting $u^*_{i,n} = \omega_1 \times |\widetilde{u}_{i,n}|$ with probability $(1-\tau)$ and $u^*_{i,n} = \omega_2 \times |\widetilde{u}_{i,n}|$ with probability τ, where $\omega_1 = 2(1-\tau)$ and $\omega_2 = -2\tau$.

(3) Construct the bootstrap sample $\{y^*_{i,n}, \sum_{j\neq i} w_{ij,n} y^*_{j,n}, \mathbf{x}_{i,n}, \mathbf{z}_{i,n};\ i = 1, \ldots, n\}$, where $y^*_{i,n}$ is generated from the restricted model under the appropriate null:

$$\mathbf{y}^*_n = \begin{cases} [\mathbf{I}_n - \widetilde{\rho}_\tau \mathbf{W}_n]^{-1} \left(\mathbf{X}_n \widetilde{\boldsymbol{\beta}}_\tau + [\mathbf{i}_n, \mathbf{Z}_n]\widetilde{\boldsymbol{\delta}}_\tau + \mathbf{u}^*_n\right) & \text{for } \text{H}_0\text{(i)} \\ [\mathbf{I}_n - \widetilde{\rho}_\tau \mathbf{W}_n]^{-1} \left(\mathbf{X}_n \widetilde{\boldsymbol{\beta}}_\tau + \mathbf{i}_n \widetilde{\delta}_\tau + \mathbf{u}^*_n\right) & \text{for } \text{H}_0\text{(ii)}, \end{cases} \tag{3.6}$$

where $\mathbf{y}^*_n = (y^*_{1,n}, \ldots, y^*_{n,n})'$ and $\mathbf{u}^*_n = (u^*_{1,n}, \ldots, u^*_{n,n})'$.

(4) Reestimate both the restricted and unrestricted models using the bootstrap sample from step (3) to obtain bootstrap residuals $\{\widetilde{u}^*_{i,n};\ i = 1, \ldots, n\}$ and $\{\widehat{u}^*_{i,n};\ i = 1, \ldots, n\}$ under H_0 and H_1, respectively.

(5) Compute the bootstrap test statistic $T^*_n = \left(RSC^*_{0,\tau} - RSC^*_{1,\tau}\right)/RSC^*_{1,\tau}$, where $RSC^*_{0,\tau} = \sum_{i=1}^{n} \zeta_\tau\{\widetilde{u}^*_{i,n}\}$ and $RSC^*_{1,\tau} = \sum_{i=1}^{n} \zeta_\tau\{\widehat{u}^*_{i,n}\}$.

[5] Feng et al. (2011) show that a traditional wild bootstrap procedure is invalid for quantile estimators due to nonlinear score functions associated with the check-function-based objective function. Alternatively, Sun (2006) introduces a modified wild bootstrap method applicable to testing in the quantile regression framework.

(6) Repeat steps (2)–(5) B times. Use the empirical distribution of $B + 1$ bootstrap statistics, where the first bootstrap test statistic equals the test statistic calculated from the raw data, to obtain the upper $a \times 100$th percentile value c_a for a given $a \in (0, 1)$. Use this c_a to approximate the upper percentile (critical) value of the test statistic T_n under H_0. We wil reject H_0 if the bootstrap test statistic is greater than c_a.

Monte Carlo simulations (discussed in Appendix B) show that the bootstrap T_n test has quite an accurate size and exhibits superb power which rises with the sample size, as expected.

4 Data

Our data come from Delaware County Auditor's Office and were obtained in the form of ArcGIS parcel shapefiles. Each parcel record contains information about house and other property characteristics such as house and lot size, number of rooms, etc. (see Table 1 for a full self-descriptive list of variables). Based on land-use codes, we retain only records containing arm's length single-family home transactions. We do so because hedonic models require competitive housing markets with buyers and sellers whose willingnesses to pay and accept are formed based on property characteristics only. Our operational sample includes 5,500 sale transactions that took place in the county during the 2009:1–2011:3 period (roughly, two years).

There are four rock mines in the county, three of which are no longer operational. All are surface mines. They were located from geographic coordinates of parcels owned by the mining companies (Ohio Department of Natural Resources, 2010, 2011) and were further verified using Google Earth. The only operational mine (state mine number: Del-5) also happens to be the largest of all by an order of magnitude. It is located in the Southwestern part of the county near the city of Delaware and is about 510 acres large,[6] which is almost triple the size of an average farm in the county (187 acres). In the case of Delaware County, all mines are limestone (but colloquially called gravel mines) and thus are subject to dynamite blasting which creates a far greater nuisance than other types of mines such as composite mines. Given that other mines in the county were no longer in operation by the period of our study and hence did not generate noise, dust and traffic, in our analysis we solely focus on the operational Del-5 mine, which is not only very large but is also located in an area of high urban growth.

Because our data are explicitly georeferenced, we use a standard software routine to calculate straight-line distances from each property to the mine centroid. This distance proxies environmental amenity associated with rock mining, with better quality occurring at farther distances from mines. We opt for such a measure over the alternative measures of environmental quality associated with disamenities such as the number of disamenities within a certain distance of a property because, in our case, we have a single occurrence of a large disamenity spread widely throughout the area. Further, since our econometric model allows environmental impacts to be nonlinear, the use of straight-line distances as a measure of environmental quality does not appear that problematic.

We also match our data with the neighborhood-specific demographic variables at the Census block level from the U.S. Census Bureau. Specifically, we include the black[7] population share, median income and the property tax rate in the neighborhood. We use these variables as observable controls for neighborhood characteristics (in addition to the spatial lag term as discussed in the introduction). We opt for these continuous measures of neighborhood characteristics over discrete

[6]Based on Google Earth Pro measurements.

[7]Variables for other non-white population groups have been consistently found to be insignificant, and their exclusion has affected the results in no material way.

Table 1. Data Summary Statistics

Variable	Units	Mean	5th Perc.	Median	95th Perc.
House Price	thousands $	258.42	64.00	232.49	552.50
Distance to Rock Mine	thousands ft.	49.12	12.92	51.14	80.27
Square Footage	ft.^{2}	2,452.99	1,188.00	2,360.00	4,054.05
Acreage	acres	0.78	0.15	0.30	3.18
Age	years	20.42	0	10	108
Story Height	cardinal number	1.79	1	2	2
# of Bedrooms	cardinal number	3.58	3	4	4
# of Bathrooms	cardinal number	2.95	1	3	5
# of Fireplaces	cardinal number	0.83	0	1	1
Garage Capacity	cardinal number	1.29	0	2	3
Attached Garage	binary indicator	0.551			
Full Basement	binary indicator	0.447			
Partial Basement	binary indicator	0.457			
Attic	binary indicator	0.095			
Central A/C	binary indicator	0.885			
Black Population Share	% pt.	3.27	0.00	1.88	11.11
Median Income	thousands $	80.04	36.40	81.20	113.00
Property Tax Rate	% pt.	1.87	1.39	1.92	2.23

The last three variables are at the Census block group level.

locality fixed effects primarily out of computational considerations because quantile estimation is known to performs rather poorly in the presence of multiple binary covariates.

5 Empirical Results

We estimate the hedonic house valuation function in the form of our PLSQAR model in (2.1), where we let the distance to nearby rock mine enter the function nonparametrically as a "z" variable with the rest of hedonic attributes included parametrically as "x" variables. All right-hand-side covariates appear in levels except for square footage and acreage to which we apply the logarithmic transformation. In the case of the number of bedrooms, bathrooms and age, we also include quadratic terms. Following the literature, we take the logarithm of the left-hand-side house price (the "y" variable) thereby facilitating the interpretation of marginal effects in terms of percentages, allowing for nonlinearities and ensuring the outcome variable can take any real value.

Given the highly uneven distribution of houses in space, we use a distance-based k-nearest-neighbor type of spatial weighting matrices to model spatial relationship across properties. The latter helps ensure that each house gets neighbors whose prices are deemed "relevant" (by getting relatively large weights) in predicting its value. The use of alternative distance-based weighting matrices, where the spatial weights are decaying functions of distance, leads to an undesirable situation when houses in highly urbanized localities have multiple "relevant" neighbors that are assigned large weights and houses in a sparsely populated countryside hardly have any such "relevant" neighbors, which obviously is inaccurate because appraisers are willing to look far for comparable properties when valuating houses in rural areas. We select the number of nearest neighbors that minimizes the AIC criterion for the median model. The data favor $k = 5$, which we use throughout.

When estimating the model, we approximate the unknown nonparametric intercept function $\alpha_{\tau,0}(\cdot)$ via cubic B-spline sieves, the order of approximation for which (in this case, the number of knots) is also selected by minimizing AIC. Throughout, we use spatial lags of continuous house-specific attributes (log square footage and log acreage) as our instruments. We do not include lags of other exogenous attributes into the instrument set because they are discrete and lead to severe multicollinearity and convergence problems.

Since the objective of our paper is to assess property-value-suppressing effects of rock mines on nearby property (and in order to conserve space), in what follows we primarily focus on the results concerning the relationship between a house's price and its distance from the mine. Consistent with the notion that rock mines are an environmental disamenity that creates negative externalities such as dust, noise and additional traffic, our expectation is the *positive* relationship between the two variables implying that the houses located farther from mines would be appraised at higher values. (The results pertaining to other house attributes are relegated to Appendix C.)

As discussed earlier, most studies pursuing the housing-market-based valuation of adverse welfare effects of environmental disamenities estimate a linear hedonic price function, which rather restrictively assumes constant marginal impact of the disamenity on house prices. Few papers that do explore potential nonlinearities have largely favored a quadratic form (e.g., Kohlhase, 1991; Hite et al., 2001) which, given its reliance on an *a priori* functional form assumption, is still subject to potential misspecification. We circumvent these problems by letting the distance between the house and a rock mine (z) enter the house valuation function in a nonparametric fashion [through an unspecified intercept function $\alpha_{\tau,0}(\cdot)$] thereby accommodating any potential nonlinearities in the relationship between (log) property values and the distance to the mine. We first examine the sensitivity of empirical results to potential functional-form misspecification of $\alpha_{\tau,0}(\cdot)$. To do so, in addition to our semiparametric PLSQAR model of house prices, we also estimate a *fully* parametric SQAR model under the following two specifications of the intercept function: (i) $\alpha_{\tau,0}(z) = a_{0,\tau} + a_{1,\tau}z + a_{2,\tau}z^2$ and (ii) $\alpha_{\tau,0}(z) = a_{0,\tau} + a_{1,\tau}z$. These specifications imply quadratic and linear functional forms of the relationship between the log price and z, respectively. Comparing the results from our flexible PLSQAR model, which lets the data determine the shape of $\alpha_{\tau,0}(\cdot)$, to those from a parametric model under these two specifications enables us to empirically assess the extent to which the hedonic estimates of property-value-suppressing effects of rock mines on nearby houses are sensitive to "correct" functional form specification of the house price function. Such a comparison is especially interesting given the wide popularity of linear and quadratic parameterizations in the literature. The parametric model under both specifications of $\alpha_{\tau,0}(\cdot)$ is estimated via a two-step procedure following Su & Yang (2011). To conserve space, we focus on the median quantile ($\tau = 0.50$) when comparing these alternative models.

Figure 1 plots the estimated intercept function across the three models. Our preferred PLSQAR model, which estimates $\alpha_{\tau,0}(z)$ nonparametrically, points to a rather steep relationship between the house price and its distance to the mine when the house is located in a close vicinity from a mine (smaller values of z) with a diminishing gradient that ultimately plateaus at around a 10-mile mark.[8] Such a shape is remarkably consistent with one's expectation that the property-value effects of environmental disamenities are a *local* phenomenon and that rock mines would not impact values of *distant* properties (with larger values of z). The latter can also be seen from Figure 2, which graphs the estimated gradient of the intercept function along with its 95% confidence bounds. The figure is indicative of a significant positive effect of z on the log house price within roughly a 10-mile radius of the mine that eventually decreases to a statistically insignificant gradient.

Comparing our model to its parametric alternatives, we expectedly find that parametric models are more susceptible to a functional-form misspecification. While the quadratic model does successfully find a decreasing gradient of $\alpha_{\tau,0}(z)$ in a close proximity from the mine, it is however unable

[8]Just above $z = 50$ thousand feet.

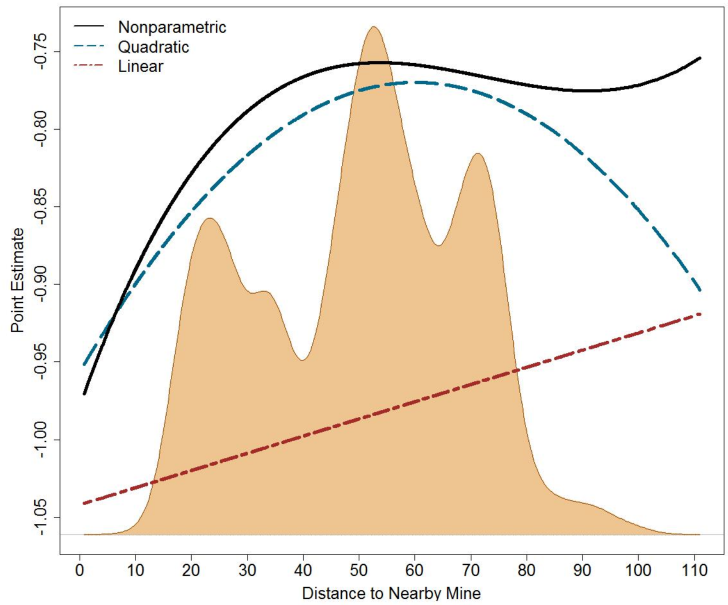

Figure 1. Estimated Intercept Functions of the Distance to Rock Mine for the Conditional Median Model [*Note:* Shaded is the kernel density of the distance variable]

to detect that rock mines appear to become rather irrelevant for the (median) price of houses lying outside their 10-mile radius zone. In fact, a parabolic relationship estimated by the quadratic model rather counter-intuitively suggests a negative (and statistically significant) relationship between the two for large values of z [see Figures 1 and 2]. This illustrates the sensitivity of parametric models (due to their inflexibility) to the inclusion of data on properties that are located farther from the disamenities and thus are less, if at all, impacted by negative environmental externalities they generate. To avoid this problem, researchers employing parametric specifications therefore usually have to prespecify a spatial radius of potential impact around the disamenity (e.g., Nelson et al., 1992; Reichert et al., 1992; Hite et al., 2001). However, such an *a priori* choice of the radius is oftentimes *ad hoc* in nature; whereas our model, owing to its nonparametric approach to modeling the distance to disamenity, essentially detects the radius of non-zero impact directly from the data. Lastly, fitting a linear SQAR model mitigates the problem but at a cost of producing a linear relationship characterized by a rather misleading "average" gradient. The latter can be vividly seen in Figure 2 which shows that, due to its inherent inability to allow for nonlinearities and hence heterogeneity across units, the linear SQAR model tends to grossly under-estimate the gradient.

However, the gradient estimates of $\alpha_{\tau,0}(z)$ plotted in Figure 2 cannot be interpreted as representing marginal partial effects of z on (median) house prices due to the appearance of spatial lag of house prices on the right-hand side of the estimated quantile function. Hence, to obtain partial effects, we consider a reduced form of the fitted outcome variable at the τth quantile: $\widehat{\mathbf{y}}_\tau = [\mathbf{I}_n - \widehat{\rho}_\tau \mathbf{W}_n]^{-1} \left(\mathbf{X}_n \widehat{\boldsymbol{\beta}}_\tau + \widehat{\boldsymbol{\alpha}}_\tau(\mathbf{Z}_n)\right)$, from where we have the following $n \times n$ matrices of marginal effects:

$$\frac{\partial \widehat{\mathbf{y}}_\tau}{\partial \mathbf{Z}_n'} = [\mathbf{I}_n - \widehat{\rho}_\tau \mathbf{W}_n]^{-1} \times \operatorname{diag}\left\{\frac{\partial \widehat{\alpha}_\tau(z_{1,n})}{\partial z_{1,n}}, \ldots, \frac{\partial \widehat{\alpha}_\tau(z_{n,n})}{\partial z_{n,n}}\right\}, \tag{5.1}$$

$$\frac{\partial \widehat{\mathbf{y}}_\tau}{\partial \mathbf{x}_{j,n}'} = [\mathbf{I}_n - \widehat{\rho}_\tau \mathbf{W}_n]^{-1} \times \widehat{\beta}_{\tau,j} \qquad \forall\ j = 1, \ldots, d_x, \tag{5.2}$$

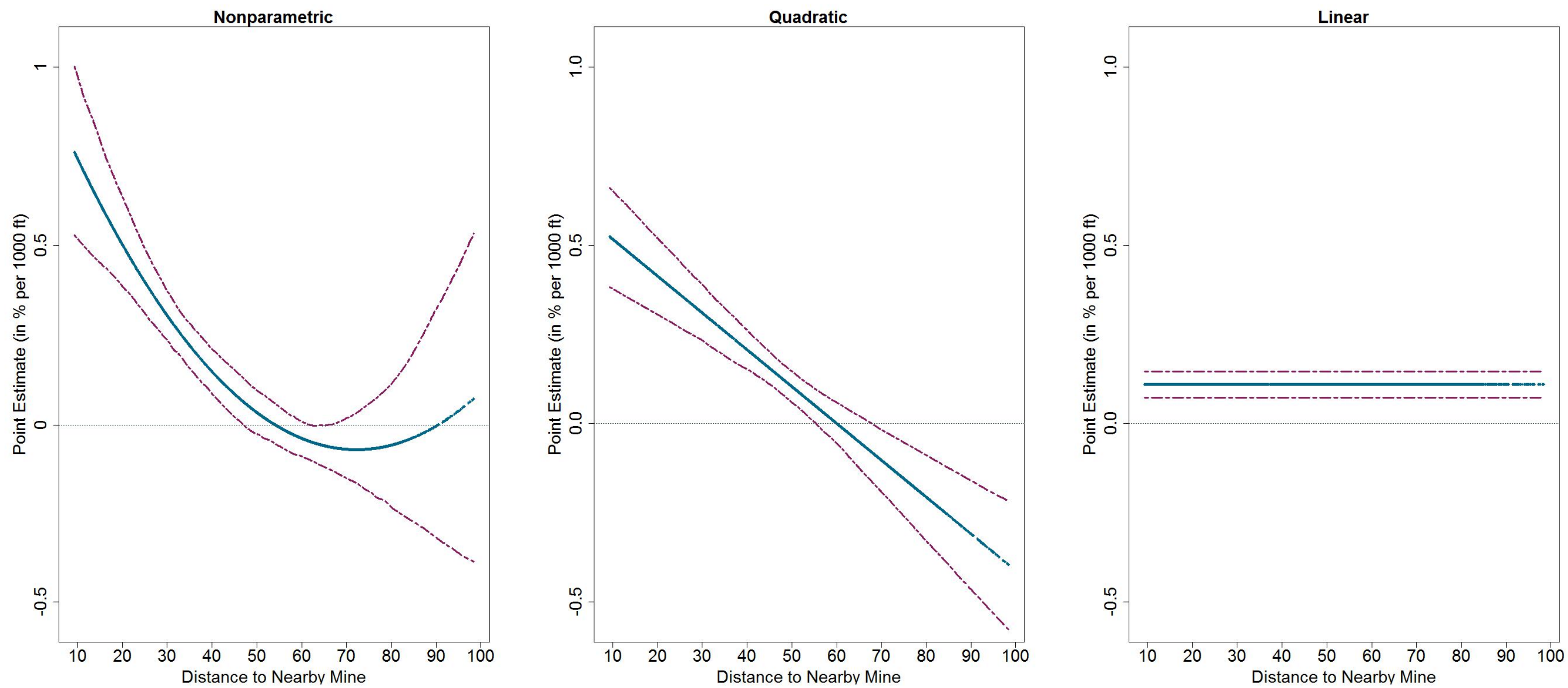

Figure 2. Estimated Gradients of Intercept Functions of the Distance to Rock Mine for the Conditional Median Model (with the 95% bootstrap confidence bounds)

Table 2. Summary of Statistically Significant Point Estimates of ME of the Distance to Rock Mine on Conditional Median of Property Value

	Entire Sample			*Within 10-Mile Radius*		
	TME	DME	IME	TME	DME	IME
			Nonparametric			
5th Perc.	–0.0853	–0.0597	–0.0257	0.1192	0.0831	0.0363
25th Perc.	0.1477	0.1037	0.0433	0.2946	0.2030	0.0885
50th Perc.	0.4629	0.3243	0.1396	0.5810	0.4046	0.1751
75th Perc.	0.8023	0.5581	0.2403	0.8560	0.5957	0.2575
95th Perc.	1.0740	0.7520	0.3227	1.0793	0.7566	0.3245
Mean	0.4836	0.3379	0.1456	0.5768	0.4031	0.1737
			Quadratic			
5th Perc.	–0.3221	–0.2263	–0.0943	0.1271	0.0897	0.0372
25th Perc.	–0.1506	–0.1071	–0.0439	0.2044	0.1449	0.0599
50th Perc.	0.1836	0.1300	0.0535	0.4338	0.3065	0.1272
75th Perc.	0.5108	0.3572	0.1508	0.6130	0.4332	0.1789
95th Perc.	0.7199	0.5063	0.2110	0.7395	0.5226	0.2167
Mean	0.1964	0.1386	0.0577	0.4146	0.2929	0.1217
			Linear			
5th Perc.	0.1646	0.1113	0.0505	0.1646	0.1113	0.0506
25th Perc.	0.1646	0.1124	0.0508	0.1646	0.1113	0.0506
50th Perc.	0.1646	0.1131	0.0514	0.1646	0.1131	0.0515
75th Perc.	0.1646	0.1137	0.0521	0.1646	0.1137	0.0522
95th Perc.	0.1646	0.1140	0.0533	0.1646	0.1140	0.0533
Mean	0.1646	0.1129	0.0516	0.1646	0.1129	0.0517

The reported estimates are in % per 1,000 ft.

where $\mathbf{x}_{j,n} = (x_{j,1}, \ldots, x_{j,n})'$ is the jth column of $\mathbf{X}_n$. In the spirit of LeSage & Pace (2009), we refer to the diagonal elements of the gradient matrices of $\widehat{\mathbf{y}}_\tau$ in (5.1)–(5.2) as direct marginal effects (DMEs) and to the off-diagonal elements as indirect marginal effects (IMEs). We analyze marginal effects row-by-row which implies a "*to* a house" interpretation, i.e., how the change in a given covariate across *all* houses affects the price of the ith house. Hence, the summation of elements in the ith row of the gradient matrices in (5.1)–(5.2) provides a measure of the total marginal effect (TME) on the ith house. Also note that, because by design the maximum-eigenvalue-standardized k-nearest-neighbor spatial weights matrix employed in the estimation is in fact row-stochastic, TMEs of covariates that have *constant* gradients (i.e., all "x" variables and, in the case of a linear parametric SQAR model, also variable z) are the same across all observations and are equal to the corresponding gradient times $(1 - \widehat{\rho}_\tau)^{-1}$.

The point estimates of total, direct and indirect marginal effects of the distance to nearby mine onto the median (log) house price across the three models are summarized in Table 2. Given that insignificant estimates are statistically indistinguishable from zero (implying no effect), here and henceforth, we focus on statistically significant estimates of marginal effects only. For inference within each model, we use the 95% bootstrap percentile confidence bounds.[9] As expected, the results are starkly different across the models, with parametric specifications consistently under-estimating the magnitude of marginal effects of the distance to rock mine on the property value. When considering the entire sample, we find that, in part due to the presence of a large number of

[9]We use 499 bootstrap replications throughout.

houses for which negative marginal effects were estimated, the quadratic model produces estimates of marginal effects on median house values that, on average, are about 59% smaller than those obtained from our semiparametric PLSQAR model. The results from a linear model are even more timid (smaller by 66% on average). Focusing on the more economically relevant results confined to a 10-mile radius zone around rock mines, we find that our PLSQAR model suggests the average TME of the distance to the mine on median house prices at around 0.57% per 1,000 feet, 0.40% points of which are the direct effect. The quadratic and linear models however yield significantly smaller estimates with the corresponding average TMEs of about 0.42% and 0.17% per 1,000 feet, which are 28% and 71% smaller than their nonparametric counterpart, respectively. The marked difference across our semiparametric model and its two parametric alternatives is apparent not only at the average values of marginal effects but along their entire distributions across houses.

Our comparison of the results from the proposed semiparametric model and those from its two parametric counterparts, until now, have largely been casual. However, given that both the linear and quadratic specifications are the special cases of our PLSQAR model, we can formally discriminate between the models by means of a specification test described in Section 3. Namely, both parametric median SQAR models can be cast as restricted models under the null of the first type $\mathrm{H}_0(\mathrm{i})$ given in (3.2) to be tested against our unrestricted PLSQAR model. We reject the null in favor of our proposed model in both cases with the bootstrap p-value no larger than 0.032. We also entertain a third specification for the parametric SQAR model whereby $\alpha_{\tau,0}(z) \equiv \alpha_{0,\tau}$ for all z, which effectively assumes that z is an irrelevant hedonic attribute that has no effect on the house price. This "constant in z" model serves an auxiliary purpose and is estimated solely in order to facilitate the test of overall relevancy of the house's proximity to a rock mine for its value. In terms of the types of null hypothesis described in Section 3, this restricted model falls under the second type of nulls $\mathrm{H}_0(\mathrm{ii})$ given in (3.3), which we test against our PLSQAR model. The corresponding bootstrap p-value is 0.038 suggesting that the proximity to rock mines *does* matter for residential property values.

Given the data lend strong support to our more flexible semiparametric model of house prices, in what follows, we therefore report the results from our PLSQAR model only. Furthermore, in the light of our earlier findings, we focus on the results confined to a local 10-mile radius zone around the mine (2,956 observations) which appear to be the most economically relevant.[10]

Table 3 summarizes statistically significant (house-specific) point estimates of marginal effects of the distance to nearby rock mine on the 0.25th, 0.50th, 0.75th and 0.95th conditional quantiles of the house price from our PLSQAR model. (We caution the reader against confusing quantiles τ of the house price distribution for which model is estimated with quantiles of the fitted distribution of observation-specific marginal effects for *each* τ.) By looking at different quantiles of the house value distribution, we are able to investigate the potentially heterogeneous impact of rock mining on residential property of *different values* thereby looking beyond the results for properties of a "typical" value delivered by standard conditional mean models. Given the tendency of quantile models to be noisier when fitted far in the tails of the distribution, in our analysis we therefore primarily focus on the interquartile range of the conditional house price distribution (setting $\tau = \{0.25, 0.50, 0.75\}$) which should give us sufficient insights into distributional effects, if any, of rock mines on house prices. That said, motivated by the proposition oftentimes claimed in the literature whereby environmental disamenities have significantly larger effects on expensive upscale properties (Reichert et al., 1992; Gayer, 2000), we also estimate our model at the 0.95th quantile to examine if the negative effects of rock mines are especially amplified when located near the most expensive houses. Overall, the results in Table 3 lend strong support to heterogeneous distributional value-

[10] To improve accuracy and to achieve better convergence rates, we still use the full sample during the estimation.

Table 3. Summary of Statistically Significant Semiparametric Estimates of ME of the Distance to Rock Mine on Conditional Quantiles of Property Value within 10-Mile Radius

	TME	DME	IME	TME	DME	IME
	0.25th Q. of Property Value			**0.75th Q. of Property Value**		
25th Perc.	0.3252	0.2182	0.1057	0.3565	0.2676	0.0887
50th Perc.	0.4781	0.3221	0.1571	0.6788	0.5105	0.1688
75th Perc.	0.5645	0.3803	0.1839	0.9979	0.7457	0.2491
Mean	0.4442	0.2993	0.1450	0.6493	0.4875	0.1618
	0.50th Q. of Property Value			**0.95th Q. of Property Value**		
25th Perc.	0.2946	0.2030	0.0885	0.5150	0.3893	0.1256
50th Perc.	0.5810	0.4046	0.1751	0.9952	0.7505	0.2437
75th Perc.	0.8560	0.5957	0.2575	1.3304	1.0048	0.3268
Mean	0.5768	0.4031	0.1737	0.9739	0.7354	0.2385

Reported are the estimates (in % per 1,000 ft) from the PLSQAR model.

suppressing effects of rock mines on the prices of nearby houses, the magnitude of which increase with the value of these houses, as expected. This distributional heterogeneity in the marginal effects can be seen even more vividly in Figure 3 which plots the distribution of the TME estimates across quantiles of the house price distribution. The figure also points to an increase in variability (i.e., a higher degree of heterogeneity across individual houses) of the TME estimates as house prices rise.

As we move from the first to third quartile of the house price distribution, we find that the average estimate of TME of the distance to nearby rock mine on house prices significantly increases from 0.44% to 0.65% per 1,000 feet [see Table 3]. When we focus on the most expensive properties at the 0.95th quantile, the TME goes up even further with the corresponding *median* estimate of about 1% and a half of point estimates being even larger than that; the mean estimate is 0.97% per 1,000 feet. For residential property in the middle of the price distribution ($\tau = 0.50$), our estimates suggest that, between two identical houses, the one located a mile closer to a rock mine is predicted to be priced, on average, at about 3.1% discount.[11] The analogous average discounts for houses in the first and third quartiles of price distribution are around 2.3 and 3.4%, respectively. For upscale property in the 0.95th quantile, it is at an astounding 5.1%. This is rather expected because of income sorting whereby higher income households have higher ability to pay for better environmental quality: in this case, distance from a disamenity. Conversely, households with lower incomes and less expensive homes are perhaps more willing to substitute environmental quality for other, more necessary, house characteristics. As a back-of-the-envelope welfare calculation using unconditional sample quantiles of house values corresponding to the fitted quantile functions,[12] the above discount estimates imply the average loss in property value associated with the house being located a mile closer to a rock mine ranging from $3,691 to $10,970 for houses within the interquartile range of price distribution. For more expensive neighborhoods in the 0.95th quantile, such losses can be, on average, as high as $28,410. We can further extend the welfare analysis to obtain aggregate property value losses due to the houses' proximity to rock mine by applying the estimated discounts to actual house prices at each observation in order to predict increase in each property's value if it were moved from its actual location to a (counterfactual) 10-mile distance from

[11] 5.28 thousand feet times the mean estimate of 0.58% per 1,000 feet. The average discount estimates for other quantiles of house price are obtained similarly.

[12] And assuming a constant marginal willingness to pay.

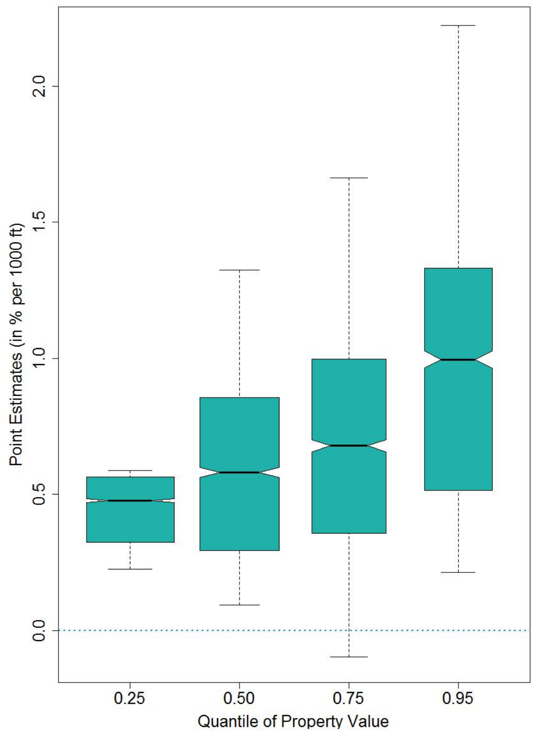

Figure 3. Statistically Significant Semiparametric Estimates of TME of the Distance to Rock Mine on Conditional Quantiles of Property Value within 10-Mile Radius across Quantiles

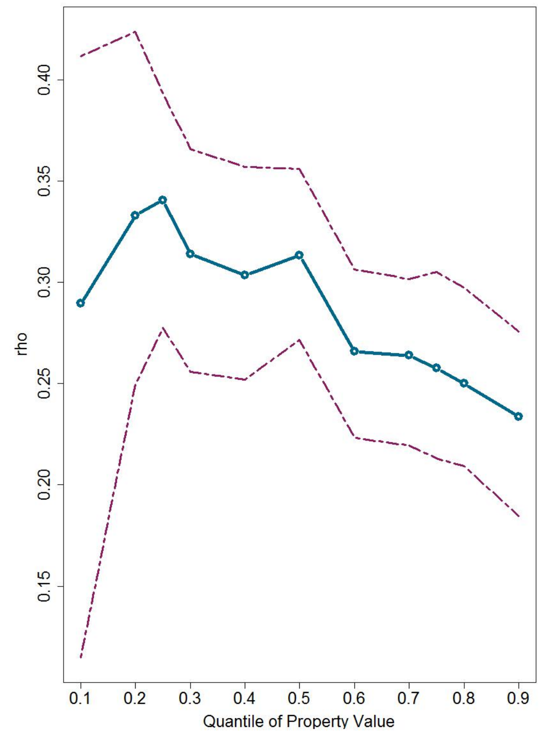

Figure 4. Semiparametric Estimates of the SAR Parameter across Quantiles (with the 95% bootstrap confidence bounds)

the mine. Applying this method to properties with statistically significant total marginal effects[13] of the distance lying within a 10-mile radius from the mine, we find a total property value loss of \$68.4 million at the median, which would have a significant impact on public goods expenditures in the county, especially on schools, because of lost tax revenue amounting to approximately \$1.3 million per annum.

Our estimates of marginal effects also indicate a decreasing (relative) importance of IMEs for residential properties of higher values. While the indirect effects working through neighbors, on average, contribute 37.8% to the TME of z_i on the log house price at the first quartile of the property value distribution, their average contribution falls quite dramatically to 26.6% for the houses at the third quartile. A plausible explanation for this is that less expensive properties may have very different interior quality levels resulting in more unobserved heterogeneity as compared to higher priced houses. Thus, in more expensive neighborhoods, the adverse effects of nearby rock mines are "priced in" directly during the valuation as opposed to via a spillover comparison to neighboring properties. In other words, we find that spatial dependence in house prices decreases as the value of property rises. To see this, consider the estimates of spatial autoregressive parameter which measures spatial dependence in the data. We summarize the estimates of $\rho_{\tau,0}$, along with their confidence bounds, across different τ of the conditional house price distribution in Figure

[13]Thereby conservatively assuming that the value of houses with insignificant marginal effects of the distance wound not increase.

4. It is evident that the SAR coefficient declines as we move from the left to the right tail of the distribution implying that neighborhood effects are more pronounced in less expensive areas. This result is similar to Liao & Wang's (2012), who estimate a fully parametric hedonic quantile model (however, with no environmental disamenities considered) and also find that the spatial autoregressive parameter declines between the 30th and 70th quantiles. Nonetheless, our estimated spatial effects are statistically significant throughout the entire house price distribution thereby indicating that the failure to account for spatial dependence, as usually done in the literature on housing-market-based valuations of adverse effects of environmental disamenities, would likely yield inconsistent estimates. This substantiates our spatial-econometric approach to hedonic modeling.

6 Conclusion

This paper provides the first estimates of the effects of rock mining—an environmental disamenity—on local residential property values. We estimate the relationship between a house's price and its distance from nearby rock mine in Delaware County, Ohio. We improve upon the conventional approach to valuating adverse effects of environmental disamenities based on hedonic house price functions by developing a novel (semiparametric) partially linear spatial quantile autoregressive model which accommodates unspecified nonlinearities, distributional heterogeneity as well as provides a means to indirectly control for unobservable house and neighborhood characteristics using the spatial dependence in the data. Our model constitutes a practically useful fusion of semi/nonparametric quantile methods with models of spatial dependence. We estimate it via a two-step nonparametric sieve IV quantile estimator. We also propose a model specification test.

We find statistically and economically significant property-value-suppressing effects of being located near an operational rock mine which gradually decline to insignificant near-zero values at a roughly ten-mile distance. Our estimates suggest that, *ceteris paribus*, a house located a mile closer to a rock mine is priced, on average, at about 2.3–5.1% discount, with more expensive properties being subject to larger markdowns. As a back-of-the-envelope welfare calculation, the above discount estimates imply the average loss in property value associated with the house being located a mile closer to a rock mine ranging from \$3,691 to \$10,970 for houses within the interquartile range of price distribution. For more expensive neighborhoods in the 0.95th quantile, such losses can be, on average, as high as \$28,410. Applying the estimated statistically significant discounts to house prices at each observation lying within a 10-mile radius from the mine to predict an increase in each property's value if it were moved from its actual location to a (counterfactual) 10-mile distance from the mine, we find the aggregate property value loss associated with rock mining in the area to be \$68.4 million at the median.

Appendix

A Brief Mathematical Proofs

For any $x \neq 0$ and y, we have

$$\zeta_\tau\{x-y\} - \zeta_\tau\{x\} = y\varphi_\tau\{y\} + \int_0^y \left(\mathbb{I}\{x \le t\} - \mathbb{I}\{x \le 0\}\right) dt, \tag{A.1}$$

where $\varphi_\tau\{u\} = \tau - \mathbb{I}\{u < 0\}$.

Lemma 1 *(i) Under Assumption* **3**, *we have* $\sup_{n,\boldsymbol{l}(i)\in D_n}\sum_{\boldsymbol{l}(j)\in D_n,\varrho(i,j)>s}|g_{ij,n}|\leq Ms^{-c_2d}$; *(ii) under Assumptions* **2–3**, $\{v_{i,n}(\rho),\boldsymbol{l}(i)\in D_n\}$ *is uniformly* L_2*-NED on* $\{\varepsilon_{in},\boldsymbol{l}(i)\in D_n\}$ *with the NED coefficients of* $\psi(s)=O(s^{-\varsigma})$; *(iii)*

$$\frac{1}{n}\sum_{i=1}^{n}\left\{f_{v_{i,n}(\rho)}\left(\eta_{i,n}(\rho)\right)-\mathbb{E}\left[f_{v_{i,n}(\rho)}\left(\eta_{i,n}(\rho)\right)\right]\right\}\mathcal{X}_{i,n}\mathcal{X}_{i,n}'\ \xrightarrow{p}\ \mathbf{0}_{1+d_x+d_m+L_n},\tag{A.2}$$

where $\eta_{i,n}(\rho)=(\rho-\rho_{\tau,0})\sum_{j=1}^{n}g_{ij,n}\left[\mathbf{x}_{j,n}'\beta_{\tau,0}+\alpha_{\tau,0}(\mathbf{z}_{j,n})\right]+\mathbf{x}_{i,n}'\left[\boldsymbol{\beta}_\tau(\rho)-\boldsymbol{\beta}_{\tau,0}\right]+\alpha_\tau^*(\mathbf{z}_{i,n},\rho)-\alpha_{\tau,0}(\mathbf{z}_{i,n})+\mathbf{m}_{i,n}'\boldsymbol{\gamma}_\tau(\rho)$ *and* $\alpha_\tau^*(\mathbf{z}_{i,n},\rho)=\boldsymbol{\phi}_{L_n}(\mathbf{z}_{i,n})'\mathcal{A}_\tau(\rho)$.

Proof. (i) Under Assumption **3**, we have $\mathbf{G}_n\equiv\mathbf{W}_n\mathbf{S}_n^{-1}=\mathbf{W}_n\sum_{k=0}^{\infty}(\rho_{\tau,0}\mathbf{W}_n)^k=\mathbf{W}_n+\rho_{\tau,0}\mathbf{W}_n^2+\rho_{\tau,0}^2\mathbf{W}_n^3+\ldots$, and hence we have

$$\begin{aligned}g_{ij,n} &= w_{ij,n}+\rho_{\tau,0}\sum_{l\neq i}w_{il,n}w_{lj,n}+\rho_{\tau,0}^2\sum_{l_2\neq i}w_{il_2,n}\left(\sum_{l_1\neq l_2}w_{l_2l_1,n}w_{l_1j,n}\right)+\rho_{\tau,0}^3\sum_{l_3\neq i}w_{il_3,n}\sum_{l_2\neq l_3}w_{l_3l_2,n}\\ &\quad\times\left(\sum_{l_1\neq l_2}w_{l_2l_1,n}w_{l_1j,n}\right)+\ldots+\rho_{\tau,0}^k\sum_{l_k\neq i}\sum_{l_{k-1}\neq l_k}\cdots\sum_{l_1\neq l_2}w_{il_k,n}w_{l_kl_{k-1},n}\ldots w_{l_2l_1,n}w_{l_1j,n}+\ldots.\end{aligned}$$

For all j such that $\boldsymbol{l}(j)\in D_n$ and $\varrho(i,j)>s$, we have

$$\begin{aligned}\sum_{\boldsymbol{l}(j)\in D_n,\varrho(i,j)>s}|g_{ij,n}| &\leq c_1\sum_{k=1}^{\infty}\rho_{\tau,0}^{k-1}\left(\frac{s}{k}\right)^{-c_2d}\leq\frac{c_1s^{c_2d}}{\rho_{\tau,0}}\int_1^\infty\rho_{\tau,0}^x x^{[c_2d]+1}dx\\ &= -\frac{c_1s^{-c_2d}}{(\ln\rho_{\tau,0})^{[c_2d]+2}}\sum_{k=0}^{[c_2d]+1}\frac{([c_2d]+1)!}{([c_2d]+1-k)!}(-\ln\rho_{\tau,0})^{[c_2d]+1-k},\end{aligned}$$

where $[a]$ is the largest integer smaller than $a>0$. This completes the proof of (i).

(ii) By definition, $v_{i,n}(\rho)=u_{i,n}+(\rho_{\tau,0}-\rho)\sum_{j=1}^{n}g_{ij,n}u_{j,n}$. Applying Minkowski's and conditional Jensen's inequalities yields

$$\begin{aligned}\|v_{i,n}(\rho)-\mathbb{E}[v_{i,n}(\rho)|\mathcal{F}_{i,n}(s)]\|_2 &\leq \|u_{i,n}-\mathbb{E}[u_{i,n}|\mathcal{F}_{i,n}(s)]\|_2+|\rho_{\tau,0}-\rho|\sum_{j=1}^{n}|g_{ij,n}|\,\|u_{j,n}-\mathbb{E}[u_{j,n}|\mathcal{F}_{i,n}(s)]\|_2\\ &\leq M\psi(s)+2|\rho_{\tau,0}-\rho|\sum_{\{j:\varrho(i,j)>s\}}|g_{ij,n}|\,\|u_{j,n}\|_2.\end{aligned}$$

This completes the proof of (ii).

(iii) Given the above results, applying Theorem 1 in Jenish & Prucha (2012) yields (A.2). ∎

Proof of Theorem 1. Denote $\widehat{\boldsymbol{\vartheta}}_\tau(\rho)=\sqrt{n}\left[\widehat{\boldsymbol{\theta}}_\tau(\rho)-\boldsymbol{\theta}_{\tau,0}(\rho)\right]$, $Y_{i,n}^*(\rho)=y_{i,n}-\rho\sum_{j\neq i}w_{ij,n}y_{j,n}-\mathcal{X}_{i,n}'\boldsymbol{\theta}_{\tau,0}(\rho)=v_{i,n}(\rho)-\eta_{i,n}$ and $Y_{i,n}(\rho)=Y_{i,n}^*(\rho)-n^{-1/2}\mathcal{X}_{i,n}'\boldsymbol{\vartheta}_\tau(\rho)$. Then, for any given $\rho\in\Lambda_\rho$, $\widehat{\boldsymbol{\vartheta}}_\tau(\rho)$ minimizes

$$Q_n(\boldsymbol{\vartheta}_\tau(\rho))=\frac{1}{n}\sum_{i=1}^{n}\left(\zeta_\tau\{Y_{i,n}(\rho)\}-\zeta_\tau\left\{Y_{i,n}^*(\rho)\right\}\right),\tag{A.3}$$

which is convex in $\boldsymbol{\vartheta}_\tau(\rho)$. We can show that, under Assumptions **2** and **5**,

$$\Pr\left[\sum_{i=1}^{n}\mathbb{I}\left\{Y_{i,n}^{*}(\rho)=0\right\}=O(1)\right]=1\quad\text{almost surely over all }\rho\in\Lambda_\rho. \tag{A.4}$$

We consider

$$Q_n(\boldsymbol{\vartheta}_\tau(\rho))=\mathbb{E}\left[Q_n(\boldsymbol{\vartheta}_\tau(\rho))\right]+\frac{\boldsymbol{\vartheta}_\tau(\rho)'}{n^{3/2}}\sum_{i=1}^{n}\mathcal{X}_{i,n}\left(\varphi_\tau\left\{Y_{i,n}^{*}(\rho)\right\}-\mathbb{E}\left[\varphi_\tau\left\{Y_{i,n}^{*}(\rho)\right\}\right]\right)+R_n(\vartheta_\tau(\rho)).$$

Denoting $t_{i,n}=n^{-1/2}\mathcal{X}_{i,n}'\boldsymbol{\vartheta}_\tau(\rho)$ and applying (A.1) and (A.4), we obtain

$$\begin{aligned}
\mathbb{E}\left[Q_n(\boldsymbol{\vartheta}_\tau(\rho))\right] &= \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}\left[\zeta_\tau\left\{Y_{i,n}^{*}(\rho)-n^{-1/2}\mathcal{X}_{i,n}'\boldsymbol{\vartheta}_\tau(\rho)\right\}-\zeta_\tau\left\{Y_{i,n}^{*}(\rho)\right\}\right]\\
&\approx \frac{\boldsymbol{\vartheta}_\tau(\rho)'}{n^{3/2}}\sum_{i=1}^{n}\mathcal{X}_{i,n}\mathbb{E}\left[\varphi_\tau\left\{Y_{i,n}^{*}(\rho)\right\}\right]+\frac{1}{n}\sum_{i=1}^{n}\int_0^{t_{i,n}}\mathbb{E}\left[\mathbb{I}\left\{Y_{i,n}^{*}(\rho)\le t\right\}-\mathbb{I}\left\{Y_{i,n}^{*}(\rho)\le 0\right\}\right]dt\\
&= \frac{\boldsymbol{\vartheta}_\tau(\rho)'}{n^{3/2}}\sum_{i=1}^{n}\mathcal{X}_{i,n}\mathbb{E}\left[\varphi_\tau\left\{Y_{i,n}^{*}(\rho)\right\}\right]+\frac{1}{n}\sum_{i=1}^{n}\int_0^{t_{i,n}}\left[F_{v_{i,n}(\rho)}(\eta_{i,n}(\rho)+t)-F_{v_{i,n}(\rho)}(\eta_{i,n}(\rho))\right]dt\\
&= \frac{\boldsymbol{\vartheta}_\tau(\rho)'}{n^{3/2}}\sum_{i=1}^{n}\mathcal{X}_{i,n}\mathbb{E}\left[\varphi_\tau\left\{Y_{i,n}^{*}(\rho)\right\}\right]+\\
&\quad \frac{\boldsymbol{\vartheta}_\tau(\rho)'}{2n^2}\sum_{i=1}^{n}f_{v_{i,n}(\rho)}(\eta_{i,n}(\rho))\mathcal{X}_{i,n}\mathcal{X}_{i,n}'\boldsymbol{\vartheta}_\tau(\rho)+O_p\left(\left(\frac{L_n}{\sqrt{n}}\right)^{3/2}\right),
\end{aligned}$$

where $F_{v_{i,n}(\rho)}(\eta_{i,n}(\rho)+t)-F_{v_{i,n}(\rho)}(\eta_{i,n}(\rho))=f_{v_{i,n}(\rho)}(\eta_{i,n}(\rho))\,t+f_{v_{i,n}(\rho)}'(\bar{\eta}_{i,n}(\rho))\,t^2/2$ with $\bar{\eta}_{i,n}(\rho)$ lying between $\eta_{i,n}(\rho)$ and $\eta_{i,n}(\rho)+t$, and

$$\left|\frac{1}{n}\sum_{i=1}^{n}\int_0^{t_{i,n}}f_{v_{i,n}(\rho)}'(\bar{\eta}_{i,n}(\rho))\,t^2dt\right|\le\frac{M}{3n^{5/2}}\sum_{i=1}^{n}\left|\mathcal{X}_{i,n}'\boldsymbol{\vartheta}_\tau(\rho)\right|^3=O_p\left(n^{-3/2}L_n^3\right)=o_p(1)$$

under Assumptions **5–6**.

Next, we consider $R_n(\boldsymbol{\vartheta}_\tau(\rho))=n^{-1}\sum_{i=1}^{n}\left(Q_{i,n}(\rho)-\mathbb{E}\left[Q_{i,n}(\rho)\right]\right)$, where

$$\begin{aligned}
Q_{i,n}(\rho) &= \zeta_\tau\left\{Y_{i,n}(\rho)\right\}-\zeta_\tau\left\{Y_{i,n}^{*}(\rho)\right\}-n^{-1/2}\boldsymbol{\vartheta}_\tau(\rho)'\mathcal{X}_i\varphi_\tau\left\{Y_{i,n}^{*}(\rho)\right\}\\
&= \int_0^{t_{i,n}}\left[\mathbb{I}\left\{v_{i,n}(\rho)\le\eta_{i,n}(\rho)+t\right\}-\mathbb{I}\left\{v_{i,n}(\rho)\le\eta_{i,n}(\rho)\right\}\right]dt.
\end{aligned}$$

Since $Q_{i,n}(\rho)$ is a function of $v_{i,n}(\rho)$, $\{Q_{i,n}(\rho),\boldsymbol{l}(i)\in D_n\}$ is uniformly L_2-NED on $\{\varepsilon_{i,n},\boldsymbol{l}(i)\in D_n\}$ with the same NED mixing coefficients as those for $\{v_{i,n}(\rho),\boldsymbol{l}(i)\in D_n\}$. It is readily seen that $\sum_{s=1}^{\infty}s^{d-1}\psi(s)\le M\sum_{s=1}^{\infty}s^{d-\varsigma-1}<M$ because $\varsigma>d$, and $\mathbb{E}\left[|Q_{i,n}(\rho)|^{2+\delta}\right]\le\mathbb{E}\left[|t_{i,n}|^{2+\delta}\right]\le Mn^{-(2+\delta)/2}L_n^{2+\delta}\to 0$ for any $\delta>0$ as $n\to\infty$ under Assumption **6**. By Lemma A.3(a) in Jenish & Prucha (2012), we obtain $\mathbb{V}\text{ar}\left[R_n(\boldsymbol{\vartheta}_\tau(\rho))\right]\le ML_n^3/n^{3/2}$ under Assumption **2(iii)**. Hence, we obtain $R_n(\boldsymbol{\vartheta}_\tau(\rho))=O_p\left((L_n/\sqrt{n})^{3/2}\right)$.

Combining the above results gives

$$Q_n(\boldsymbol{\vartheta}_\tau(\rho))=\frac{\boldsymbol{\vartheta}_\tau(\rho)'}{n^{3/2}}\sum_{i=1}^{n}\mathcal{X}_{i,n}\varphi_\tau\left\{Y_{i,n}^{*}(\rho)\right\}+\frac{1}{2n}\boldsymbol{\vartheta}_\tau(\rho)'\ \boldsymbol{\Sigma}_\tau(\rho)\ \boldsymbol{\vartheta}_\tau(\rho)+o_p(1) \tag{A.5}$$

under Assumption **6**, and this result holds uniformly over $\rho \in \Lambda_\rho$ by the convexity lemma of Pollard (1991), where

$$\boldsymbol{\Sigma}_\tau(\rho) = \lim_{n\to\infty} n^{-1} \sum_{i=1}^{n} \mathbb{E}\left[f_{v_{i,n}(\rho)}\left(\eta_{i,n}(\rho)\right)\right] \mathcal{X}_{i,n}\mathcal{X}_{i,n}' \tag{A.6}$$

by Lemma **1(iii)**. It then follows that

$$\widehat{\boldsymbol{\vartheta}}_\tau(\rho) = -\frac{\boldsymbol{\Sigma}_\tau^{-1}(\rho)}{\sqrt{n}} \sum_{i=1}^{n} \mathcal{X}_{i,n}\varphi_\tau\left\{Y_{i,n}^*(\rho)\right\} + o_p(1) \tag{A.7}$$

holds uniformly over $\rho \in \Lambda_\rho$. So, we obtain

$$\sqrt{n}\left[\widehat{\boldsymbol{\theta}}_\tau(\rho) - \boldsymbol{\theta}_{\tau,0}(\rho)\right] = -\frac{\boldsymbol{\Sigma}_\tau^{-1}(\rho)}{\sqrt{n}} \sum_{i=1}^{n} \left[\tau - \mathbb{I}\left\{v_{i,n}(\rho) \leq \eta_{i,n}(\rho)\right\}\right] \mathcal{X}_{i,n} + o_p(1). \tag{A.8}$$

Applying Lemma **1(ii)** and the CLT of Jenish & Prucha (2012, Theorem 2), we obtain that $n^{-1/2}\sum_{i=1}^{n}\left(\mathbb{I}\left\{v_{i,n}(\rho) \leq \eta_{i,n}(\rho)\right\} - \mathbb{E}\left[\mathbb{I}\left\{v_{i,n}(\rho) \leq \eta_{i,n}(\rho)\right\}\right]\right)\mathcal{X}_{i,n} = O_p(1)$ element by element, where $n^{-1}\sum_{i=1}^{n}\left\{\tau - \mathbb{E}\left[\mathbb{I}\left\{v_{i,n}(\rho) \leq \eta_{i,n}(\rho)\right\}\right]\right\}\mathcal{X}_{i,n} = 0$ since this term is the first-order condition of $\max_{\boldsymbol{\vartheta}_\tau(\rho)} \mathbb{E}\left[Q_n\left(\boldsymbol{\vartheta}_\tau(\rho)\right)\right]$. Hence, under Assumption **5(iii)**, we obtain $\left\|\widehat{\boldsymbol{\theta}}_\tau(\rho) - \boldsymbol{\theta}_{\tau,0}(\rho)\right\| = O_p\left(\sqrt{L_n/n}\right)$ uniformly over ρ. This completes the proof of this theorem. ■

Proof of Theorem 2. In Step 2, we calculate $\widehat{\rho}_\tau = \arg\min_\rho \widehat{\boldsymbol{\gamma}}_\tau(\rho)'\mathbf{V}_n\widehat{\boldsymbol{\gamma}}_\tau(\rho)$, where $\widehat{\boldsymbol{\gamma}}_\tau(\rho) = \boldsymbol{\gamma}_{\tau,0}(\rho) + o_p(1)$ uniformly over ρ by Theorem **1**. Since $\widehat{\boldsymbol{\gamma}}_\tau(\rho)$ is continuous in ρ and $\boldsymbol{\gamma}_{\tau,0}(\rho)'\mathbf{V}_n\boldsymbol{\gamma}_{\tau,0}(\rho)$ has a minimum value at $\rho_{\tau,0}$, we obtain $\widehat{\rho}_\tau \xrightarrow{p} \rho_{\tau,0}$ by Theorem 2.1 in Newey & McFadden (1994). Since $\boldsymbol{\theta}_{\tau,0}(\rho)$ is continuous in ρ, we have $\left\|\widehat{\boldsymbol{\theta}}_\tau - \boldsymbol{\theta}_{\tau,0}\right\| = o_p(1)$.

When $\rho = \rho_{\tau,0}$, we have $v_{i,n}(\rho_{\tau,0}) = u_{i,n}$, $\eta_{i,n}(\rho_{\tau,0}) = \alpha_{\tau,0}^*(\mathbf{z}_{i,n}) - \alpha_{\tau,0}(\mathbf{z}_{i,n})$, and

$$\boldsymbol{\Sigma}_\tau = \boldsymbol{\Sigma}_\tau(\rho_{\tau,0}) = \lim_{n\to\infty} n^{-1}\sum_{i=1}^{n} f_{u_{i,n}}(0)\,\mathcal{X}_i\mathcal{X}_i' \quad \text{by (2.15) and (A.6).}$$

Let ρ_n be a constant satisfying $\rho_n = \rho_{\tau,0} + o(1)$, and denote $\widehat{Y}_{i,n}(\rho_n) = y_{i,n} - \rho_n\sum_{j\neq i} w_{ij,n}y_{j,n} - \mathcal{X}_{i,n}'\widehat{\boldsymbol{\theta}}_\tau(\rho_n) = Y_{i,n}^*(\rho_n) + \mathcal{X}_{i,n}'\left(\boldsymbol{\theta}_{\tau,0}(\rho_n) - \widehat{\boldsymbol{\theta}}_\tau(\rho_n)\right)$. By Lemma A.2 in Ruppert & Carroll (1980), we have $o_p(1) = n^{-1/2}\sum_{i=1}^{n}\varphi_\tau\left\{\widehat{Y}_{i,n}(\rho_n)\right\}\mathcal{X}_{i,n}$. Let $\chi_{i,n}(\rho,\boldsymbol{\theta}) = \varphi_\tau\left\{y_{i,n} - \rho\sum_{j\neq i} w_{ij,n}y_{j,n} - \mathcal{X}_{i,n}'\boldsymbol{\theta}(\rho)\right\}\mathcal{X}_{i,n}$ and $\mathbb{E}\left[\chi_{i,n}\left(\rho_n, \widehat{\boldsymbol{\theta}}_\tau(\rho_n)\right)\right] = \mathbb{E}\left[\chi_{i,n}(\rho,\boldsymbol{\theta}(\rho))\right]_{(\rho,\boldsymbol{\theta}(\rho)) = \left(\rho_n, \widehat{\boldsymbol{\theta}}_\tau(\rho_n)\right)}$ and decompose

$$\begin{aligned}
\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\varphi_\tau\left\{\widehat{Y}_{i,n}(\rho_n)\right\}\mathcal{X}_{i,n} &= \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left[\chi_{i,n}\left(\rho_n, \widehat{\boldsymbol{\theta}}_\tau(\rho_n)\right) - \mathbb{E}\left[\chi_{i,n}\left(\rho_n, \widehat{\boldsymbol{\theta}}_\tau(\rho_n)\right)\right]\right] \\
&\quad + \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\mathbb{E}\left[\chi_{i,n}\left(\rho_n, \widehat{\boldsymbol{\theta}}_\tau(\rho_n)\right)\right].
\end{aligned}$$

First, since $\mathbb{E}\left[\chi_{i,n}(\rho, \boldsymbol{\theta}_\tau(\rho))\right] = \mathbb{E}\left[\tau - \mathbb{I}\left\{v_{i,n}(\rho) \leq \eta_{i,n}(\rho)\right\}\right]\mathcal{X}_{i,n}$, we have

$$\mathbb{E}\left[\tau - \mathbb{I}\left\{v_{i,n}(\rho) \leq \eta_{i,n}(\rho)\right\}\right] = \mathbb{E}\left[F_{u_{i,n}}(0|\bar{u}_{i,n}) - F_{u_{i,n}}\left(\frac{(\rho_{\tau,0} - \rho)\,\bar{u}_{i,n} + \eta_{i,n}(\rho)}{1 + (\rho_{\tau,0} - \rho)\,g_{ii,n}}\,\middle|\,\bar{u}_{i,n}\right)\right]$$

$$= \quad -\mathbb{E}\left[\bar{u}_{i,n} f_{u_{i,n}}\left(\bar{c}_{i,n}|\bar{u}_{i,n}\right)\right] \frac{\rho_{\tau,0}-\rho}{1+\left(\rho_{\tau}-\rho\right) g_{ii,n}}$$
$$-\mathbb{E}\left[f_{u_{i,n}}\left(\bar{c}_{i,n}|\bar{u}_{i,n}\right)\right] \frac{\eta_{i,n}(\rho)}{1+\left(\rho_{\tau,0}-\rho\right) g_{ii,n}}$$

if $1+\left(\rho_{\tau,0}-\rho\right) g_{ii,n}>0$, where $\bar{c}_{i,n}$ lies between 0 and $\left[\left(\rho_{\tau,0}-\rho\right) \bar{u}_{i,n}+\eta_{i,n}(\rho)\right]\left[1+\left(\rho_{\tau,0}-\rho\right) g_{ii,n}\right]^{-1}$. Therefore, if $\lim_{n\to\infty} \inf_{1\leq i\leq n}\left[1+\left(\rho_{\tau,0}-\rho_n\right) g_{ii,n}\right]=c_g>0$, we obtain

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} \mathbb{E}\left[\chi_{i,n}\left(\rho_n, \widehat{\boldsymbol{\theta}}_{\tau}\left(\rho_n\right)\right)\right] \approx -\mathcal{A}_1 \sqrt{n}\left(\rho_{\tau,0}-\rho_n\right)- \mathcal{A}_2 \sqrt{n}\left(\boldsymbol{\theta}_{\tau,0}\left(\rho_n\right)-\widehat{\boldsymbol{\theta}}_{\tau}\left(\rho_n\right)\right),$$

where

$$\mathcal{A}_1 = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n}\left[1+\left(\rho_{\tau,0}-\rho_n\right) g_{ii,n}\right]^{-1} \mathbb{E}\left[f_{u_{i,n}}\left(0|\bar{u}_{i,n}\right)\left(\bar{u}_{i,n}+\sum_{j=1}^{n} g_{ij,n}\left[\mathbf{x}_{j,n}^{\prime} \boldsymbol{\beta}_{\tau,0}+\alpha_{\tau,0}\left(\mathbf{z}_{j,n}\right)\right]\right)\right] \mathcal{X}_{i,n},$$
$$\mathcal{A}_2 = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n}\left[1+\left(\rho_{\tau,0}-\rho_n\right) g_{ii,n}\right]^{-1} \mathbb{E}\left[f_{u_{i,n}}\left(0|\bar{u}_{i,n}\right)\right] \mathcal{X}_{i,n} \mathcal{X}_{i,n}^{\prime}. \tag{A.9}$$

Second, Jenish (2016) has proven the stochastic equicontinuity result of an empirical process for the smooth function of a NED spatial process and finite parameters. Applying Theorem 5 in Jenish (2016), we obtain that the equicontinuity result also holds for $\Delta_n(\rho, \boldsymbol{\theta}(\rho))$ here, i.e.,

$$\left\|\Delta_n\left(\rho_n, \widehat{\boldsymbol{\theta}}_{\tau}\left(\rho_n\right)\right)-\Delta_n\left(\rho_{\tau,0}, \boldsymbol{\theta}_{\tau,0}\left(\rho_{\tau,0}\right)\right)\right\|=o_p(1), \tag{A.10}$$

where

$$\mathcal{A}_{n,0} \equiv \Delta_n\left(\rho_{\tau,0}, \boldsymbol{\theta}_{\tau,0}\left(\rho_{\tau,0}\right)\right)=\frac{1}{\sqrt{n}} \sum_{i=1}^{n}\left(\varphi_{\tau}\left\{Y_{i,n}^*\left(\rho_{\tau,0}\right)\right\}-\mathbb{E}\left[\varphi_{\tau}\left\{Y_{i,n}^*\left(\rho_{\tau,0}\right)\right\}\right]\right) \mathcal{X}_{i,n}. \tag{A.11}$$

Third, combining the above results yields

$$\sqrt{n}\left(\widehat{\boldsymbol{\theta}}_{\tau}\left(\rho_n\right)-\boldsymbol{\theta}_{\tau,0}\left(\rho_n\right)\right)=-\mathcal{A}_2^{-1} \mathcal{A}_{n,0}+\mathcal{A}_2^{-1} \mathcal{A}_1 \sqrt{n}\left(\rho_{\tau,0}-\rho_n\right).$$

Partition below matrix/vector conformably with 1, 2 and 3 corresponding to $\mathbf{x}_{i,n}$, $\mathbf{m}_{i,n}$ and $\boldsymbol{\phi}_{L_n}\left(\mathbf{z}_{i,n}\right)$, respectively:

$$\mathcal{A}_1=\left[\begin{array}{c}\mathcal{A}_{1,1} \\ \mathcal{A}_{1,2} \\ \mathcal{A}_{1,3}\end{array}\right], \quad \mathcal{A}_2^{-1}=\left[\begin{array}{ccc}\mathcal{A}_2^{11} & \mathcal{A}_2^{12} & \mathcal{A}_2^{13} \\ \mathcal{A}_2^{21} & \mathcal{A}_2^{22} & \mathcal{A}_2^{23} \\ \mathcal{A}_2^{31} & \mathcal{A}_2^{32} & \mathcal{A}_2^{33}\end{array}\right]=\left[\begin{array}{c}\mathcal{A}_2^{1} \\ \mathcal{A}_2^{2} \\ \mathcal{A}_2^{3}\end{array}\right] \text{ and } \mathcal{A}_{n,0}=\left[\begin{array}{c}\mathcal{A}_{n,0,1} \\ \mathcal{A}_{n,0,2} \\ \mathcal{A}_{n,0,3}\end{array}\right].$$

Then, we have

$$\sqrt{n}\left(\begin{array}{c}\widehat{\boldsymbol{\beta}}_{\tau}\left(\rho_n\right)-\boldsymbol{\beta}_{\tau,0}\left(\rho_n\right) \\ \widehat{\boldsymbol{\gamma}}_{\tau}\left(\rho_n\right)-\boldsymbol{\gamma}_{\tau,0}\left(\rho_n\right)\end{array}\right)=-\left[\begin{array}{c}\mathcal{A}_2^{1} \\ \mathcal{A}_2^{2}\end{array}\right] \mathcal{A}_{n,0}+\left[\begin{array}{c}\mathcal{A}_2^{1} \\ \mathcal{A}_2^{2}\end{array}\right] \mathcal{A}_1 \sqrt{n}\left(\rho_{\tau,0}-\rho_n\right). \tag{A.12}$$

In addition, from Step 2, we have $\widehat{\rho}_{\tau}=\arg\min_{\rho_n} \widehat{\boldsymbol{\gamma}}_{\tau}^{\prime}\left(\rho_n\right) \mathbf{V}_n \widehat{\boldsymbol{\gamma}}_{\tau}\left(\rho_n\right)$. Applying the CLT of Jenish & Prucha (2012) gives

$$\mathbf{e}_j^{\prime} \mathcal{A}_{n,0} \xrightarrow{d} \mathbb{N}\left(\mathbf{0}, \mathbf{e}_j^{\prime} \boldsymbol{\Omega}_{\tau} \mathbf{e}_j\right)$$

with

$$\mathbf{\Omega}_\tau = \lim_{n\to\infty} n^{-1}\sum_{i=1}^{n}\sum_{j=1}^{n}\mathcal{X}_{i,n}\mathbb{C}\text{ov}\left[\varphi_\tau\left\{u_{i,n}\right\},\varphi_\tau\left\{u_{j,n}\right\}\right]\mathcal{X}_{jn}' = O\left(1\right)$$

by Lemma A.3 in Jenish & Prucha (2012), where $\mathbf{e}_j$ is any one of the column vectors of $\mathbf{I}_{d_x+d_m+L_n}$. It follows that $\sqrt{n}\left\|\widehat{\boldsymbol{\gamma}}_\tau\left(\rho_n\right)-\boldsymbol{\gamma}_{\tau,0}\left(\rho_n\right)\right\| = O_p\left(1\right)$, which implies that $\sqrt{n}\left(\rho_{\tau,0}-\rho_n\right)=O\left(1\right)$. Consequently, we obtain

$$\sqrt{n}\left(\widehat{\rho}_\tau-\rho_{\tau,0}\right)=\left[\mathcal{A}_1'\left(\mathcal{A}_2^2\right)'\mathbf{V}_n\mathcal{A}_2^2\mathcal{A}_1\right]^{-1}\mathcal{A}_1'\left(\mathcal{A}_2^2\right)'\mathbf{V}_n\mathcal{A}_2^2\mathcal{A}_{n,0}\equiv\mathcal{M}_\rho\mathcal{A}_{n,0}.$$

Therefore we have

$$\sqrt{n}\begin{pmatrix}\widehat{\boldsymbol{\beta}}_\tau\left(\rho_n\right)-\boldsymbol{\beta}_{\tau,0}\left(\rho_n\right)\\ \widehat{\boldsymbol{\gamma}}_\tau\left(\rho_n\right)-\boldsymbol{\gamma}_{\tau,0}\left(\rho_n\right)\end{pmatrix}=\begin{bmatrix}\mathcal{A}_2^1\\ \mathcal{A}_2^2\end{bmatrix}\left(\mathcal{A}_1\mathcal{M}_\rho-\mathbf{I}_{d_x+d_m+L_n}\right)\mathcal{A}_{n,0},$$

so that we obtain

$$\sqrt{n}\mathbf{\Sigma}_n^{-1/2}\begin{bmatrix}\widehat{\rho}_\tau-\rho_{\tau,0} & \widehat{\boldsymbol{\beta}}_\tau'-\boldsymbol{\beta}_{\tau,0}' & \widehat{\boldsymbol{\gamma}}_\tau'\end{bmatrix}'\xrightarrow{d}\mathbb{N}\left(\mathbf{0},\mathbf{I}\right),$$

where $\mathbf{\Sigma}_n=\mathcal{P}\mathbf{\Omega}_\tau\mathcal{P}'$ with $\mathcal{P}=\left[\mathcal{M}_\rho',\mathbf{\Psi}'\left[\left(\mathcal{A}_2^1\right)',\left(\mathcal{A}_2^2\right)'\right]\right]'$ and $\mathbf{\Psi}=\mathcal{A}_1\mathcal{M}_\rho-\mathbf{I}_{d_x+d_m+L_n}$.

Lastly, we have

$$\sqrt{n}\left[\widehat{\alpha}_\tau\left(\mathbf{z}\right)-\alpha_{\tau,0}\left(\mathbf{z}\right)\right]=\sqrt{n}\boldsymbol{\phi}_{L_n}\left(\mathbf{z}\right)'\left(\widehat{\mathcal{A}}_\tau-\mathcal{A}_{\tau,0}\right)=\boldsymbol{\phi}_{L_n}\left(\mathbf{z}\right)'\mathcal{A}_2^3\left[\mathcal{A}_1\mathcal{M}_\rho-\mathbf{I}_{d_x+d_m+L_n}\right]\mathcal{A}_{n,0},$$

and hence obtain

$$\sqrt{n/\omega_{n,\tau}}\left[\widehat{\alpha}_\tau\left(\mathbf{z}\right)-\alpha_{\tau,0}\left(\mathbf{z}\right)\right]\xrightarrow{d}\mathbb{N}\left(0,1\right),$$

where $\omega_{n,\tau}=\boldsymbol{\phi}_{L_n}\left(\mathbf{z}\right)'\mathcal{A}_2^3\mathbf{\Psi}\mathbf{\Omega}_\tau\mathbf{\Psi}'\left(\mathcal{A}_2^3\right)'\boldsymbol{\phi}_{L_n}\left(\mathbf{z}\right)$. This completes the proof of this theorem. ■

Proof of Theorem 3. By definition, $RSC_{0,\tau}=\sum_{i=1}^n\zeta_\tau\{\widetilde{u}_{i,n}\}$ and $RSC_{1,\tau}=\sum_{i=1}^n\zeta_\tau\{\widehat{u}_{i,n}\}$ with

$$\begin{aligned}\widetilde{u}_{i,n} &= y_{i,n}-\widetilde{\rho}_\tau\sum_{j\neq i}w_{ij,n}y_{j,n}-\mathbf{x}_{i,n}'\widetilde{\boldsymbol{\beta}}_\tau-\mathbf{z}_{i,n}'\widetilde{\boldsymbol{\delta}}_\tau=Y_{i,n,0}\left(\widetilde{\rho}_\tau\right)+\mathbf{m}_i'\widetilde{\boldsymbol{\gamma}}_\tau,\\ \widehat{u}_{i,n} &= y_{i,n}-\widehat{\rho}_\tau\sum_{j\neq i}w_{ij,n}y_{j,n}-\mathbf{x}_{i,n}'\widehat{\boldsymbol{\beta}}_\tau-\widehat{\alpha}_\tau(\mathbf{z}_{i,n})=Y_{i,n}\left(\widehat{\rho}_\tau\right)+\mathbf{m}_i'\widehat{\boldsymbol{\gamma}}_\tau,\end{aligned}$$

where $Y_{i,n,0}^*\left(\rho\right)=y_{i,n}-\rho\sum_{j\neq i}w_{ij,n}y_{j,n}-\mathbf{x}_{i,n}'\boldsymbol{\beta}_{\tau,0}\left(\rho\right)-\mathbf{z}_{i,n}'\boldsymbol{\delta}_{\tau,0}\left(\rho\right)-\mathbf{m}_{i,n}'\boldsymbol{\gamma}_{\tau,0}\left(\rho\right)$, $Y_{i,n}^*\left(\rho\right)$ is defined the same as in the proof of Theorem **1** and, to simplify notation, we let $\mathbf{z}_{i,n}$ include 1 in the model under the null. Denoting $\boldsymbol{\chi}_{i,n}=\left[\mathbf{x}_{i,n}',\mathbf{z}_{i,n}',\mathbf{m}_{i,n}'\right]'$ and $Y_{i,n,0}\left(\rho\right)=Y_{i,n,0}^*\left(\rho\right)-\boldsymbol{\chi}_i'\left[\widetilde{\boldsymbol{\theta}}_\tau\left(\rho\right)-\boldsymbol{\theta}_{\tau,0}\left(\rho\right)\right]$, we have

$$\begin{aligned}n^{-1}RSC_{0,\tau} &= n^{-1}\sum_{i=1}^n\zeta_\tau\left\{Y_{i,n,0}\left(\widetilde{\rho}_\tau\right)+\mathbf{m}_{i,n}'\widetilde{\boldsymbol{\gamma}}_\tau\right\}\\ &= n^{-1}\sum_{i=1}^n\left[\zeta_\tau\left\{Y_{i,n,0}\left(\widetilde{\rho}_\tau\right)\right\}-\zeta_\tau\left\{Y_{i,n,0}^*\left(\widetilde{\rho}_\tau\right)\right\}\right]+n^{-1}\sum_{i=1}^n\zeta_\tau\left\{Y_{i,n,0}^*\left(\widetilde{\rho}_\tau\right)\right\}\ +\\ &\quad n^{-1}\sum_{i=1}^n\left[\zeta_\tau\left\{Y_{i,n,0}\left(\widetilde{\rho}_\tau\right)+\mathbf{m}_{i,n}'\widetilde{\boldsymbol{\gamma}}_\tau\right\}-\zeta_\tau\left\{Y_{i,n,0}\left(\widetilde{\rho}_\tau\right)\right\}\right]\end{aligned}$$

$$
\begin{aligned}
&= Q_{n,0}\left(\widetilde{\boldsymbol{\theta}}_\tau\left(\widetilde{\rho}_\tau\right)\right) + n^{-1}\sum_{i=1}^n \zeta_\tau\left\{Y^*_{i,n,0}\left(\widetilde{\rho}_\tau\right)\right\} + n^{-1}\sum_{i=1}^n\left[\zeta_\tau\left\{Y_{i,n,0}\left(\widetilde{\rho}_\tau\right)+\mathbf{m}'_{i,n}\widetilde{\boldsymbol{\gamma}}_\tau\right\} - \zeta_\tau\left\{Y_{i,n,0}\left(\widetilde{\rho}_\tau\right)\right\}\right] \\
&\approx -\frac{1}{2n^2}\left[\sum_{i=1}^n \boldsymbol{\chi}_{i,n}\varphi_\tau\left\{u_{i,n}\right\}\right]'\boldsymbol{\Sigma}^{-1}_{\tau,0}\left[\sum_{i=1}^n \boldsymbol{\chi}_{i,n}\varphi_\tau\left\{u_{i,n}\right\}\right] + n^{-1}\sum_{i=1}^n \zeta_\tau\left\{u_{i,n}\right\} + o_p\left(n^{-1}\right),
\end{aligned}
$$

where, following the proof of Theorem 1, we have $Q_{n,0}\left(\boldsymbol{\vartheta}_\tau\left(\rho\right)\right) = n^{-1}\sum_{i=1}^n\left[\zeta_\tau\left\{Y_{i,n,0}\left(\rho\right)\right\} - \zeta_\tau\left\{Y^*_{i,n,0}\left(\rho\right)\right\}\right]$, and $\boldsymbol{\Sigma}_{\tau,0} = \lim_{n\to\infty} n^{-1}\sum_{i=1}^n f_{u_{i,n}}\left(0\right)\boldsymbol{\chi}_i\boldsymbol{\chi}'_i$. In addition, we obtain

$$
\begin{aligned}
n^{-1}RSC_{1,\tau} &= n^{-1}\sum_{i=1}^n \zeta_\tau\left\{Y_{i,n}\left(\widehat{\rho}_\tau\right)+\mathbf{m}'_{i,n}\widehat{\boldsymbol{\gamma}}_\tau\right\} \\
&= Q_n\left(\widehat{\boldsymbol{\theta}}_\tau\left(\widehat{\rho}_\tau\right)\right) + n^{-1}\sum_{i=1}^n \zeta_\tau\left\{Y^*_{i,n}\left(\widehat{\rho}_\tau\right)\right\} + n^{-1}\sum_{i=1}^n\left[\zeta_\tau\left\{Y_{i,n}\left(\widehat{\rho}_\tau\right)+\ \mathbf{m}'_{i,n}\widehat{\boldsymbol{\gamma}}_\tau\right\} - \zeta_\tau\left\{Y_{i,n}\left(\widehat{\rho}_\tau\right)\right\}\right] \\
&\approx -\frac{1}{2n^2}\left[\sum_{i=1}^n \mathcal{X}_{i,n}\varphi_\tau\left\{u_{i,n}\right\}\right]'\boldsymbol{\Sigma}^{-1}_{\tau}\left[\sum_{i=1}^n \mathcal{X}_{i,n}\varphi_\tau\left\{u_{i,n}\right\}\right] + n^{-1}\sum_{i=1}^n \zeta_\tau\left\{u_{i,n}\right\} + o_p\left(n^{-1}\right)
\end{aligned}
$$

by the proof of Theorem **1.**

Therefore, under H_0, we obtain

$$
RSC_{0,\tau} - RSC_{1,\tau} \approx \left(\mathcal{B}'_{n,1}\boldsymbol{\Sigma}^{-1}_\tau\mathcal{B}_{n,1} - \mathcal{B}'_{n,0}\boldsymbol{\Sigma}^{-1}_{\tau,0}\mathcal{B}_{n,0}\right)/2,
$$

where $\mathcal{B}_{n,0} = n^{-1/2}\sum_{i=1}^n \boldsymbol{\chi}_{i,n}\varphi_\tau\left\{u_{i,n}\right\}$ and $\mathcal{B}_{n,1} = n^{-1/2}\sum_{i=1}^n \mathcal{X}_{i,n}\varphi_\tau\left\{u_{i,n}\right\}$. By Seber (2008, Property 20.17), $\mathcal{B}'_{n,1}\boldsymbol{\Sigma}^{-1}_\tau\mathcal{B}_{n,1}$ can be rewritten as a linear combination of $d_x+d_m+L_n$ independent chi-squared random variables and $\mathcal{B}'_{n,0}\boldsymbol{\Sigma}^{-1}_{\tau,0}\mathcal{B}_{n,0}$ can be rewritten as a linear combination of $d_x+d_m+d_z$ independent chi-squared random variables. Since $n^{-1}RSC_{1,\tau} = n^{-1}\sum_{i=1}^n \zeta_\tau\left\{u_{i,n}\right\} + o_p\left(1\right) \xrightarrow{p} n^{-1}\sum_{i=1}^n \mathbb{E}\left[\zeta_\tau\left\{u_{i,n}\right\}\right]$ by the LLN derived in Jenish & Prucha (2012), we obtain that

$$
T_n = \frac{RSC_{0,\tau} - RSC_{1,\tau}}{RSC_{1,\tau}} \approx \frac{\left(\mathcal{B}'_{n,1}\boldsymbol{\Sigma}^{-1}_\tau\mathcal{B}_{n,1} - \mathcal{B}'_{n,0}\boldsymbol{\Sigma}^{-1}_{\tau,0}\mathcal{B}_{n,0}\right)/2}{\sum_{i=1}^n \mathbb{E}\left[\zeta_\tau\left(u_{i,n}\right)\right]} = O_p\left(\frac{L_n}{n}\right).
$$

Under H_1, following the proof of Theorem **1**, we can show that there exists parameter $\boldsymbol{\theta}_\tau\left(\rho_\tau\right) = \left(\rho_\tau, \boldsymbol{\delta}'_\tau, \boldsymbol{\gamma}'_\tau\right)' \neq \boldsymbol{\theta}_{\tau,0}$ such that $\widetilde{\rho}_\tau - \rho_\tau = O_p\left(n^{-1/2}\right)$ and $\widetilde{\boldsymbol{\theta}}_\tau\left(\rho\right) - \boldsymbol{\theta}_\tau\left(\rho\right) = O_p\left(n^{-1/2}\right)$ uniformly over $\rho \in \Lambda_\rho$. Then, it follows that

$$
n^{-1}RSC_{0,\tau} = n^{-1}\sum_{i=1}^n \zeta_\tau\left\{Y^*_{i,n,0}\left(\widetilde{\rho}_\tau\right) + \mathbf{m}'_{i,n}\widetilde{\boldsymbol{\gamma}}_\tau\right\} \approx n^{-1}\sum_{i=1}^n \zeta_\tau\left\{Y^*_{i,n,0}\left(\widetilde{\rho}_\tau\right)\right\} = O_p\left(1\right),
$$

because $Y^*_{i,n,0}\left(\widetilde{\rho}_\tau\right) = y_{i,n} - \widetilde{\rho}_\tau\sum_{j\neq i} w_{i,nj}y_{j,n} - \mathbf{x}'_{i,n}\boldsymbol{\beta}_\tau\left(\widetilde{\rho}_\tau\right) - \mathbf{z}'_{i,n}\boldsymbol{\delta}_\tau\left(\widetilde{\rho}_\tau\right) - \mathbf{m}'_{i,n}\boldsymbol{\gamma}_\tau\left(\widetilde{\rho}_\tau\right) = u_{i,n} + \left(\rho_{\tau,0} - \widetilde{\rho}_\tau\right)\sum_{j\neq i} w_{ij,n}y_{j,n} + \mathbf{x}'_{i,n}\left(\boldsymbol{\beta}_{\tau,0} - \widetilde{\boldsymbol{\beta}}_\tau\left(\widetilde{\rho}_\tau\right)\right) + \alpha_{\tau,0}\left(\mathbf{z}_{i,n}\right) - \mathbf{z}'_{i,n}\widetilde{\boldsymbol{\delta}}_\tau\left(\widetilde{\rho}_\tau\right) - \mathbf{m}'_{i,n}\boldsymbol{\gamma}_\tau\left(\widetilde{\rho}_\tau\right) = u_{i,n} + O_p\left(1\right)$ uniformly over i. Hence, we obtain

$$
T_n = \frac{RSC_{0,\tau} - RSC_{1,\tau}}{RSC_{1,\tau}} \approx \frac{n^{-1}\sum_{i=1}^n\left[\zeta_\tau\left\{Y^*_{i,n,0}\left(\widetilde{\rho}_\tau\right)\right\} - \zeta_\tau\left\{u_{i,n}\right\}\right]}{n^{-1}\sum_{i=1}^n \mathbb{E}\left[\zeta_\tau\left\{u_{i,n}\right\}\right]} = O_p\left(1\right).
$$

■

B Monte Carlo Simulations

In this section, we evaluate the finite-sample performance of our proposed estimator and the test statistic in a small set of Monte Carlo simulations.

B.1 Estimator

We generate the data using a random-coefficient "rendition" of our model in (2.1). Specifically, our PLSQAR model can be motivated by the following random-coefficient partially linear model:

$$y_{i,n} = \rho_0^*(\upsilon_{i,n}) \sum_{j \neq i} w_{ij,n} y_{j,n} + \mathbf{x}_{i,n}' \boldsymbol{\beta}_0^*(\upsilon_{i,n}) + \alpha_0^*(\mathbf{z}_{i,n}, \upsilon_{i,n}), \tag{B.1}$$

where $\upsilon_{i,n} \perp (\mathbf{X}_n, \mathbf{Z}_n, \mathbf{M}_n)$ is the scalar random disturbance. In the structural framework, $\upsilon_{i,n}$ can be interpreted as capturing heterogeneity in the outcome variable $y_{i,n}$ due to some unobserved factors. Further, if following Chernozhukov & Hansen (2005, 2006) one were to assume that $\upsilon_{i,n} \sim$ *i.i.d.* $\mathbb{U}(0,1)$ and that the so-called structural quantile function of interest

$$q\left(\sum_{j \neq i} w_{ij,n} y_{j,n}, \mathbf{x}_{i,n}, \mathbf{z}_{i,n}, \tau\right) = \rho_0^*(\tau) \sum_{j \neq i} w_{ij,n} y_{j,n} + \mathbf{x}_{i,n}' \boldsymbol{\beta}_0^*(\tau) + \alpha_0^*(\mathbf{z}_{i,n}, \tau) \tag{B.2}$$

is such that $\partial q(\cdot, \tau)/\partial \tau > 0$, the event $\{y_{i,n} \leq \rho_0^*(\tau) \sum_{j \neq i} w_{ij,n} y_{j,n} + \mathbf{x}_{i,n}' \boldsymbol{\beta}_0^*(\tau) + \alpha_0^*(\mathbf{z}_{i,n}, \tau)\}$ becomes equivalent to the event $\{\upsilon_{i,n} \leq \tau\}$. Then, it is straightforward to establish the following quantile restriction:

$$\Pr[u_{i,n}^* \leq 0 | \mathbf{X}_n, \mathbf{Z}_n, \mathbf{M}_n] = \tau, \tag{B.3}$$

where, in an analogy to our model in (2.1), the new quantile error term is defined as $u_{i,n}^* \equiv y_{i,n} - \rho_0^*(\tau) \sum_{j \neq i} w_{ij,n} y_{j,n} - \mathbf{x}_{i,n}' \boldsymbol{\beta}_0^*(\tau) - \alpha_0^*(\mathbf{z}_{i,n}, \tau)$. Clearly, (B.1) and (B.3) are respectively analogous to (2.1) and (2.2).

Thus, we use the following process to generate the data:

$$y_i = \rho_0(\upsilon_i) \sum_{j \neq i} w_{ij} y_j + x_i \beta_0(\upsilon_i) + \alpha_0(z_i, \upsilon_i) \quad \forall\ i = 1, \ldots, n, \tag{B.4}$$

where the variables are randomly drawn as follows: $z_i \sim$ *i.i.d.* $\mathbb{U}(-1,1)$, $x_i = 0.5 z_i + \xi_i$ with $\xi_i \sim$ *i.i.d.* $\mathbb{N}(0,1)$, and $\upsilon_i \sim$ *i.i.d.* $\mathbb{U}(0,1)$. Following Kelejian & Prucha (1999) and Jin & Lee (2015), we choose a circular "1 ahead and 1 behind" structure of $\mathbf{W}_n$, where a given spatial unit is related to one neighbor immediately ahead and one neighbor immediately behind it in a row. Each of these two neighbors are assigned an equal non-zero weight of 0.5. When specifying parameter functions, we consider the following two data-generating processes:

$$\rho_{\tau,0} \equiv \rho_0(\upsilon)\big|_{\upsilon=\tau} = 0.5 + 0.15\Phi^{-1}(\upsilon) \qquad \text{[DGP \#1 \& DGP \#2]} \tag{B.5}$$

$$\beta_{\tau,0} \equiv \beta_0(\upsilon)\big|_{\upsilon=\tau} = 0.2 + 0.15\Phi^{-1}(\upsilon) \qquad \text{[DGP \#1 \& DGP \#2]} \tag{B.6}$$

$$\alpha_{\tau,0}(z) \equiv \alpha_0(z,\upsilon)\big|_{\upsilon=\tau} = \sin(1 + 1.5z) + \begin{cases} 0.15\Phi^{-1}(\upsilon) & \text{[DGP \#1]} \\ 0.15\exp\{-z^2\}\Phi^{-1}(\upsilon). & \text{[DGP \#2]} \end{cases} \tag{B.7}$$

We conduct the experiments at three different quantiles $\tau = \{0.25, 0.50, 0.75\}$ for each of which the considered sample sizes are $n = \{125, 250, 500, 1000\}$. For each τ-n pair, we simulate the model

Table B.1. Simulation Results for the Estimator

	$\boldsymbol{\tau = 0.25}$				$\boldsymbol{\tau = 0.50}$				$\boldsymbol{\tau = 0.75}$			
	$n = 125$	$n = 250$	$n = 500$	$n = 1000$	$n = 125$	$n = 250$	$n = 500$	$n = 1000$	$n = 125$	$n = 250$	$n = 500$	$n = 1000$
DGP #1												
$\rho_{\tau,0}$												
RMSE	0.09081	0.05811	0.03928	0.03009	0.07679	0.04391	0.02859	0.01985	0.09293	0.06012	0.04155	0.03029
MAE	0.07137	0.04556	0.03069	0.02390	0.05766	0.03437	0.02279	0.01604	0.07194	0.04590	0.03239	0.02375
$\beta_{\tau,0}$												
RMSE	0.04177	0.03045	0.02350	0.01875	0.03216	0.02130	0.01384	0.00957	0.04470	0.03095	0.02391	0.01870
MAE	0.03327	0.02442	0.01922	0.01602	0.02545	0.01633	0.01103	0.00755	0.03594	0.02429	0.01956	0.01614
$\alpha_{\tau,0}(z_i)$												
RMSE	0.09774	0.06333	0.04378	0.03327	0.08292	0.05015	0.03516	0.02609	0.10699	0.06757	0.04813	0.03567
MAE	0.08657	0.05511	0.03720	0.02803	0.07254	0.04297	0.02950	0.02160	0.09426	0.05938	0.04223	0.03096
DGP #2												
$\rho_{\tau,0}$												
RMSE	0.08516	0.05320	0.03979	0.03214	0.06697	0.03983	0.02548	0.01764	0.08204	0.06030	0.04401	0.03259
MAE	0.06624	0.04195	0.03135	0.02622	0.05068	0.03155	0.02020	0.01411	0.06500	0.04661	0.03396	0.02602
$\beta_{\tau,0}$												
RMSE	0.04265	0.03385	0.02706	0.02329	0.02942	0.01964	0.01359	0.00893	0.04494	0.03330	0.02731	0.02329
MAE	0.03460	0.02804	0.02320	0.02089	0.02351	0.01511	0.01092	0.00708	0.03652	0.02689	0.02354	0.02104
$\alpha_{\tau,0}(z_i)$												
RMSE	0.08686	0.05677	0.04213	0.03400	0.06861	0.04342	0.03016	0.02260	0.09372	0.06687	0.04929	0.03864
MAE	0.07716	0.04990	0.03660	0.02943	0.06023	0.03738	0.02550	0.01864	0.08333	0.05970	0.04379	0.03396

Table B.2. Simulation Results for the T_n Statistic with $\tau = 0.50$

Signif. Level	*Estimated Size* $n = 100$	$n = 200$	$n = 400$	*Estimated Power* $n = 100$	$n = 200$	$n = 400$
Case of $\mathrm{H}_0(i)$						
		DGP #1			DGP #3	
1%	0.020	0.016	0.014	0.892	0.981	1.000
5%	0.059	0.059	0.053	0.975	1.000	1.000
10%	0.122	0.106	0.094	0.993	1.000	1.000
20%	0.232	0.194	0.196	1.000	1.000	1.000
Case of $\mathrm{H}_0(ii)$						
		DGP #2			DGP #3	
1%	0.028	0.016	0.014	0.719	0.880	0.993
5%	0.085	0.070	0.070	0.941	0.996	1.000
10%	0.128	0.110	0.122	0.985	1.000	1.000
20%	0.239	0.196	0.232	0.998	1.000	1.000

Note: The reported are the rejection frequencies over 500 simulations.

500 times. We use cubic B-splines to approximate unknown function $\alpha_0(\cdot)$. For simplicity, we set $L_n = 3$ in our experiments for all sample sizes since the range of n is not that large. We compute the root mean squared error (RMSE) and the mean absolute error (MAE) for each fixed coefficient across 500 iterations. For a varying nonparametric intercept function, RMSE and MAE are first computed for each simulation iteration; reported are their averages computed over 500 iterations.

The results are reported in Table B.1. Consistent with our theory, performance of the estimator improves with an increase in the sample size across all quantiles. As one would normally expect, it performs better for "middle" quantiles (median, in our case): RMSE and MAE somewhat worsen when we estimate the model closer to tails of the response distribution.

B.2 Specification Tests

We next examine the small-sample performance of our proposed specification test statistic. To conserve space, we only consider $\tau = 0.50$. The sample sizes are $n = \{100, 200, 400\}$, and the number of simulation replications is 500. Residuals under H_1 are obtained via our proposed PLSQAR model using cubic B-splines to approximate the unknown function $\alpha_0(\cdot)$. Residuals under H_0 are obtained via Su & Yang's (2011) estimator. Given the sample size, for each simulation, we calculate our test statistic from the simulated data plus 199 bootstrap test statistics. Then, from the 200 test statistic values, we obtain the 1%, 5%, 10% and 20% upper percentile (critical) values.

To assess power and size of the test, we consider the following four experimental designs for the data-generating process given in (B.4):

(1) The null in (3.2) is true: $\rho_{\tau,0} \equiv \rho_0(\upsilon)\big|_{\upsilon=\tau} = 0.5 + 0.15\Phi^{-1}(\upsilon)$, $\beta_{\tau,0} \equiv \beta_0(\upsilon)\big|_{\upsilon=\tau} = 0.2 + 0.15\Phi^{-1}(\upsilon)$ and $\alpha_{\tau,0}(z) \equiv \alpha_0(z, \upsilon)\big|_{\upsilon=\tau} = 0.5 + 0.5z + 0.15\Phi^{-1}(\upsilon)$;

(2) The null in (3.3) is true: $\rho_{\tau,0} \equiv \rho_0(\upsilon)\big|_{\upsilon=\tau} = 0.5 + 0.15\Phi^{-1}(\upsilon)$, $\beta_{\tau,0} \equiv \beta_0(\upsilon)\big|_{\upsilon=\tau} = 0.2 + 0.15\Phi^{-1}(\upsilon)$ and $\alpha_{\tau,0}(z) \equiv \alpha_0(z, \upsilon)\big|_{\upsilon=\tau} = 0.5 + 0.15\Phi^{-1}(\upsilon)$ for all z;

(3) The alternative in (3.4) is true: $\rho_{\tau,0} \equiv \rho_0(\upsilon)\big|_{\upsilon=\tau} = 0.5 + 0.15\Phi^{-1}(\upsilon)$, $\beta_{\tau,0} \equiv \beta_0(\upsilon)\big|_{\upsilon=\tau} = 0.2 + 0.15\Phi^{-1}(\upsilon)$ and $\alpha_{\tau,0}(z) \equiv \alpha_0(z, \upsilon)\big|_{\upsilon=\tau} = \sin(1 + 1.5z) + 0.15\Phi^{-1}(\upsilon)$.

The results presented in Table B.2 show that the test has quite an accurate size across all null

Table C.1. Semiparametric Estimates of Constant Parameters on House Attributes in the Conditional Quantile Regression of Property Value across Quantiles

	Quantiles of Property Value			
	0.25th	0.50th	0.75th	0.95th
Log Sq. Footage	**0.59100**	**0.58160**	**0.59024**	**0.58871**
	(0.53217; 0.64599)	(0.53713; 0.62548)	(0.54446; 0.63067)	(0.49993; 0.74361)
Log Acreage	**0.04253**	**0.06913**	**0.08138**	**0.09038**
	(0.01883; 0.06745)	(0.04775; 0.08817)	(0.06252; 0.09893)	(0.02675; 0.11778)
Story Height	**–0.05092**	**–0.09042**	**–0.09235**	**–0.13096**
	(–0.09016; –0.00927)	(–0.11479; –0.06307)	(–0.11880; –0.06453)	(–0.18673; –0.05093)
# Bedrooms	–0.00629	–0.01029	–0.02846	–0.14829
	(–0.14271; 0.10882)	(–0.11146; 0.08000)	(–0.11103; 0.06366)	(–0.35613; 0.20943)
# Bedrooms^2	–0.00420	–0.00227	0.00006	0.01576
	(–0.02006; 0.01373)	(–0.01471; 0.01206)	(–0.01374; 0.01176)	(–0.03296; 0.04323)
# Bathrooms	0.06181	**0.06611**	0.00290	–0.03061
	(–0.00550; 0.12941)	(0.01357; 0.11258)	(–0.05774; 0.05336)	(–0.14870; 0.09881)
# Bathrooms^2	–0.00041	0.00180	**0.01322**	**0.02173**
	(–0.00877; 0.00853)	(–0.00366; 0.00784)	(0.00655; 0.02102)	(0.00243; 0.03575)
Full Basement	**0.17764**	**0.11541**	**0.10999**	0.07606
	(0.12002; 0.23109)	(0.07540; 0.15296)	(0.08254; 0.14185)	(–0.01222; 0.22164)
Partial Basement	**0.14850**	**0.07297**	**0.06104**	0.01918
	(0.09096; 0.20614)	(0.03693; 0.11070)	(0.03572; 0.09072)	(–0.06952; 0.15137)
Attic	0.02001	0.00833	**0.02287**	0.01788
	(–0.00580; 0.04998)	(–0.01016; 0.02775)	(0.00395; 0.04785)	(–0.03912; 0.08237)
Attached Garage	0.02530	0.01621	–0.03072	–0.11543
	(–0.03024; 0.07103)	(–0.01856; 0.04644)	(–0.07117; 0.00431)	(–0.23245; 0.04623)
Garage Capacity	**0.02446**	**0.02412**	**0.02613**	0.03682
	(0.00620; 0.04629)	(0.01226; 0.03812)	(0.01350; 0.04132)	(–0.02873; 0.07669)
# Fireplaces	**0.05920**	**0.05461**	**0.03577**	0.02552
	(0.03759; 0.08208)	(0.03640; 0.07530)	(0.01886; 0.05363)	(–0.02504; 0.08159)
Central A/C	**0.13311**	**0.11955**	**0.08045**	0.01313
	(0.06906; 0.19630)	(0.05463; 0.17715)	(0.03524; 0.13024)	(–0.09633; 0.11826)
Age	**–0.00603**	**–0.00464**	**–0.00258**	–0.00108
	(–0.00793; –0.00372)	(–0.00611; –0.00313)	(–0.00400; –0.00120)	(–0.00490; 0.00250)
Age^2	**0.00001**	**0.00001**	**0.00001**	0.00001
	(0.00000; 0.00003)	(0.00000; 0.00003)	(0.00000; 0.00002)	(–0.00002; 0.00003)

Reported are the estimates from a semiparametric PLSQAR model. The 95% bootstrap (percentile) confidence bounds in parentheses. Statistically significant estimates are in bold.

hypotheses regardless of n. Furthermore, the test exhibits superb power which increases with the sample size, as expected.

C Additional Results

In this section, we briefly comment on the results corresponding to hedonic attributes other than the distance to rock mine included in the estimated house price function. Their fixed parameter estimates (with bootstrap confidence bounds) across quantiles of the house price distribution are reported in Table C.1. For the estimates of median marginal effects of statistically significant covariates, see Table C.2. Among these non-distance variables, log square footage of house, log acreage and story height are the only ones consistently found to be significant across all estimated quantiles of the house price distribution. Interestingly, no other house attribute has a significant

Table C.2. Semiparametric Estimates of Median ME of Selected House Attributes on Conditional Quantiles of Property Value across Quantiles

	Quantiles of Property Value			
	0.25th	0.50th	0.75th	0.95th
Log Sq. Footage				
TME	0.8961	0.8467	0.7950	0.7882
Median DME	0.6048	0.5928	0.5976	0.5958
Median IME	0.2914	0.2540	0.1974	0.1925
Log Acreage				
TME	0.0645	0.1007	0.1096	0.1210
Median DME	0.0435	0.0705	0.0824	0.0915
Median IME	0.0210	0.0302	0.0272	0.0295
Story Height				
TME	–0.0772	–0.1316	–0.1244	–0.1753
Median DME	–0.0521	–0.0922	–0.0935	–0.1325
Median IME	–0.0251	–0.0395	–0.0309	–0.0428
Full Basement				
TME	0.2694	0.1680	0.1481	0.1018
Median DME	0.1818	0.1176	0.1114	0.0770
Median IME	0.0876	0.0504	0.0368	0.0249
Partial Basement				
TME	0.2252	0.1062	0.0822	0.0257
Median DME	0.1520	0.0744	0.0618	0.0194
Median IME	0.0732	0.0319	0.0204	0.0063
Garage Capacity				
TME	0.0371	0.0351	0.0352	0.0493
Median DME	0.0250	0.0246	0.0265	0.0373
Median IME	0.0121	0.0105	0.0087	0.0120
# Fireplaces				
TME	0.0898	0.0795	0.0482	0.0342
Median DME	0.0606	0.0557	0.0362	0.0258
Median IME	0.0292	0.0238	0.0120	0.0083
Central A/C				
TME	0.2018	0.1741	0.1084	0.0176
Median DME	0.1362	0.1219	0.0814	0.0133
Median IME	0.0656	0.0522	0.0269	0.0043

Reported are the medians of point estimates of MEs from the PLSQAR model estimated for a given conditional quantile of property value.

impact on property values in the 0.95th quantile. Houses in this top quantile include older (historic) houses in Delaware City as well as recently built McMansion-style houses. More generally, we find that the number of bedrooms and bathrooms in the house, the presence of an attic and the garage being attached to the main house are largely statistically insignificant across all quantiles which likely is due to property heterogeneity inherent with rapid urbanization. Among the statistically significant house attributes, the square footage has by far the largest marginal effect on the property value with its magnitude declining as the house price rises. We document a similar declining marginal effects (across quantiles) for the basement variables, the number of fireplaces and the presence of central air-conditioning system in the house. From Table C.2, it appears that garage capacity is equally valued by all home buyers regardless of the property value, whereas the lot size exhibits increasing importance for buyers of higher priced houses. The estimates of the total marginal effects of story height are negative across all quantiles with larger (absolute) magnitudes estimated at the higher house price quantiles. This likely is an artifact of changing consumer preferences as well as building trends in the area given that single-story houses have become more common in recent years.

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